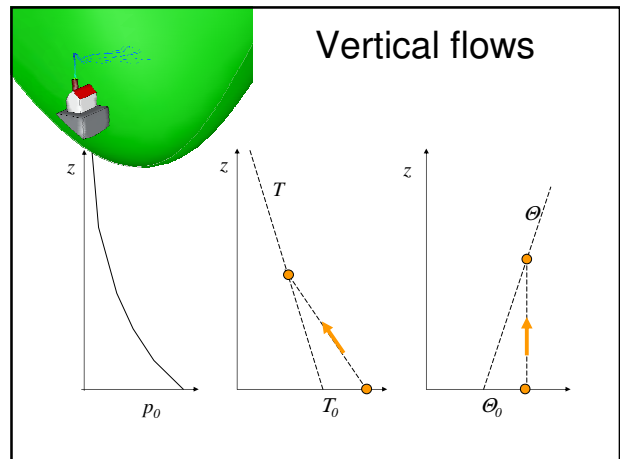


## 7. Atmospheric flows

Dr. Gergely Kristóf  
 Dept. of Fluid Mechanics, BME  
 March, 2009.



## Energy balance of a moving volume of air in the atmosphere

The atmosphere feels colder at the top of high mountain.  
 Cold air does not flow down to sea level. Why?

The cold air flow would warm up due to the increasing pressure.  
 (We cannot observe this phenomenon in any laboratory experiments.)

$$du = \delta q + \delta w \quad \text{change of the internal energy of a unit mass of air.}$$

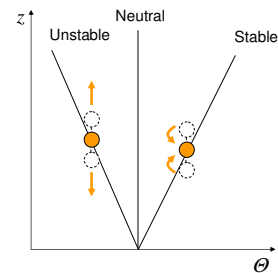
$\delta w$ : work due to:  
 - reversible compression;  
 - viscous dissipation.

$\delta q$ : heating due to:  
 - turbulent transport,  
 - radiation,  
 - latent heat release (phase change).

The change of the thermodynamic state in a rapid vertical flow of dry air is dominated by the reversible compression work, therefore it is isentropic.

## Stable and unstable stratifications

On the basis of the potential temperature profile we can analyze the atmospheric stability:



## The potential temperature: $\Theta$

For isentropic flows: 
$$\frac{T}{T_0} = \left(\frac{p}{p_0}\right)^{\frac{\kappa-1}{\kappa}} = \left(\frac{\rho}{\rho_0}\right)^{\kappa-1}$$

in which  $\kappa$  is the ratio of specific heats, thus

$$\frac{\kappa-1}{\kappa} = \frac{c_p - c_v}{c_p} = \frac{R}{c_p}$$

The potential temperature of the atmospheric air in a point characterized by temperature  $T$  and pressure  $p$  is defined as:

$$\Theta = T \left(\frac{10^5 \text{ Pa}}{p}\right)^{R/c_p}$$

$\Theta$  would be the temperature if the air parcel would taken down to sea level.

## Hydrostatics

$$\frac{\partial p}{\partial z} = -\rho g$$

In order to solve this we need to find relation between  $p$  and  $\rho$ .

Barotropic relation:

- $\rho = \text{const.}$  homogenous atm.
- $\rho = \frac{p}{RT_0}, T_0 = \text{const.}$  isothermal atm.
- $\rho = \rho_0 \left(\frac{p}{p_0}\right)^m, m = \text{const.}$  polytropic atm. e.g.  $\Theta = \text{const.}$
- $\rho = \frac{p}{RT}, \bar{T} = f(z)$

## Problem #7.1

Please, calculate the pressure profile for a given (linear) temperature profile:

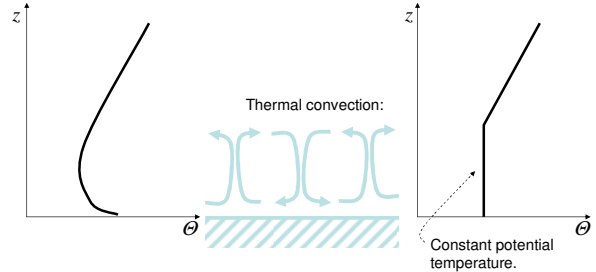
$$\bar{T} = T_0 - \gamma z$$

in which  $\gamma = -\frac{\partial T}{\partial z} = \text{const.}$

To the solution

## Unstable stratification

During the summer, when the surface is heavily heated:



## The dry adiabatic temperature gradient

We compare the  $T(p)$  relation of a linear temperature model with those of an adiabatic model.

From the linear  $T$  profil of  $\gamma$  gradient we obtained: The adiabatic relation:

$$p = p_0 \left( \frac{T_0 - \gamma z}{T_0} \right)^{\frac{g}{R\gamma}} \rightarrow \frac{T}{T_0} = \left( \frac{p}{p_0} \right)^{\frac{R\gamma}{g}} \quad \frac{T}{T_0} = \left( \frac{p}{p_0} \right)^{\frac{R}{c_p}}$$

By comparing the exponents:  $\frac{R\gamma_{adiab.}}{g} = \frac{R}{c_p}$

we obtain the adiabatic temperature gradient:  $\gamma_{adiab.} = \Gamma = \frac{g}{c_p} = \frac{9.8}{1000} = 9.8 \frac{\text{K}}{\text{km}}$

Thus the dry adiabatic  $T$  profile is a linear profile of  $\Gamma$  gradient.

## Buoyancy and the Boussinesq model

The vertical component of the equation of motion:

$$\rho \frac{dw}{dt} = -\frac{\partial p}{\partial z} - \rho g$$

We define the hydrostatic state:

$$0 = -\frac{\partial \bar{p}}{\partial z} - \bar{\rho} g$$

We decompose the profiles by using the hydrostatic state:

$$p = \bar{p} + p' \quad \rho = \bar{\rho} + \rho' \quad T = \bar{T} + T'$$

After subtracting the hydrostatic profile we obtain:

$$\rho \frac{dw}{dt} = -\frac{\partial p'}{\partial z} - \rho' g$$

The density perturbation can be expressed in terms of the cubic heat expansion coefficient  $\beta$ :

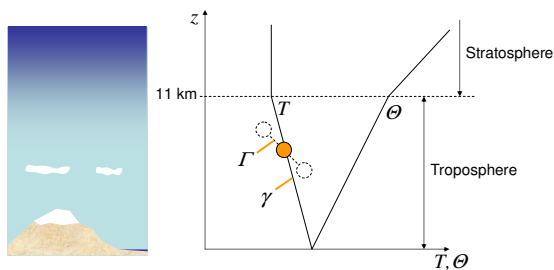
$$\rho' = -\bar{\rho} \beta T'$$

If  $p = \text{const.}$ :

$$\frac{\rho'}{\rho} = -\frac{T'}{\bar{T}}$$

$$\beta = \bar{T}^{-1}$$

## The standard atmosphere



Standard (average) tropospheric temperature gradient:

$$\gamma = 6.5 \frac{\text{K}}{\text{km}} < \Gamma$$

Reference values:

$$T_0 = 288.15 \text{ K}$$

$$p_0 = 1.01325 \cdot 10^5 \text{ Pa}$$

## Acoustic filtering

The density need to depend on the pressure, but we need to eliminate the acoustic waves. (Acoustic effects require very small time stepping.)

The continuity equation for compressible fluid:

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho u}{\partial x} + \frac{\partial \rho v}{\partial y} + \frac{\partial \rho w}{\partial z} = 0$$

$$\rho \equiv \bar{\rho}(z)$$

$$\nabla \cdot (\bar{\rho} \mathbf{v}) = 0$$

Since the average density is a function of the altitude, this is more complex continuity equation we normally use in incompressible flow models.

## Coriolis force

Inertial forces in a rotating frame:

- Centrifugal force — Taken into account in  $g$ .
- Coriolis force — Effects only moving bodies.

$$\underline{C} = -2\Omega \times \underline{v} = -2 \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \Omega_x & \Omega_y & \Omega_z \\ u & v & w \end{vmatrix}$$

$$C_x = 2v\Omega_z - 2w\Omega_y \quad \text{the average of } w \text{ is } 0.$$

$$C_y = 2w\Omega_x - 2u\Omega_z$$

$$C_z = 2u\Omega_y - 2v\Omega_x$$

the buoyancy force prevails in vertical flows

$$C_x \approx v f$$

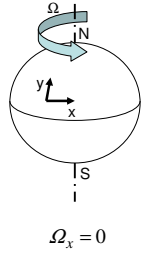
$$C_y \approx -u f$$

$$C_z \approx 0$$

in which  $f = 2\Omega \sin \phi$       $\Omega_y = \Omega \cos(\phi)$

$\Omega_z = \Omega \sin(\phi)$

$\phi$  the geographic latitude



## Problem #7.2

- a) Calculate the below non-dimensional pressure gradient for a gradient wind in cylindrical system of coordinates:

$$\frac{\partial p}{\partial r}$$

$$\rho f v_g$$

$r$  is the distance from the center of the cyclonic motion and  $v_g = f(r)$ .

- b) What is the magnitude of the non-dimensional pressure gradient for a geostrophic wind?

To the solution

## Geostrophic wind

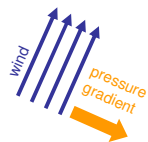
Steady flow with the assumptions below:

$$0 = -\frac{1}{\rho} \frac{\partial p}{\partial x} + f v_g$$

No convective acceleration: streamlines are approximated with parallel lines. Strain stresses are neglected.

$$0 = -\frac{1}{\rho} \frac{\partial p}{\partial y} - f u_g$$

On the North hemisphere:



$$0 = -\frac{1}{\rho} \frac{\partial p}{\partial z} - g$$

Hydrostatic equilibrium in  $z$  direction.

$$u_g \frac{\partial p}{\partial x} + v_g \frac{\partial p}{\partial y} = 0 \quad \text{thus} \quad \bar{v}_g \cdot \nabla p = 0$$

The pressure gradient is perpendicular to the direction of the equilibrium flow.

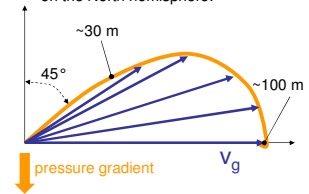
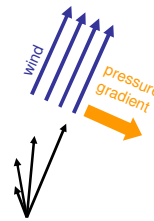
... self-consistent specification of the boundary conditions.

## Ekman spiral

Wind direction changes rapidly at the top of the boundary layer (in the outer layer). When approaching the ground the wind direction turns towards the decreasing pressure. The phenomenon is described by a model similar to the geostrophic wind model, but forces rising from the turbulent stresses must be taken into account:

$$\bar{F}_{turb} = \frac{\partial}{\partial z} \left( v_t \frac{\partial u}{\partial z} \right) \bar{i} + \frac{\partial}{\partial z} \left( v_t \frac{\partial v}{\partial z} \right) \bar{j} \quad \left[ \frac{N}{m^3} \right]$$

The solution for a medium latitude case on the North hemisphere:



## Gradient wind

Same as the geostrophic wind excepting, that **circular streamlines** are assumed.

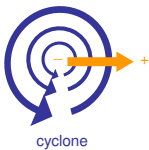
The centrifugal must be taken into account:

$$u_g \frac{\partial u_g}{\partial x} + v_g \frac{\partial u_g}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + f v_g$$

$$u_g \frac{\partial v_g}{\partial x} + v_g \frac{\partial v_g}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial y} - f u_g$$

On the North hemisphere:

$$0 = -\frac{1}{\rho} \frac{\partial p}{\partial z} - g$$



## Velocity magnitude in the boundary layer

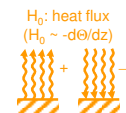
Two important effects must be taken into account: **surface roughness** and **thermal stratification**.

The Monyin-Obuhov profile:

$$\frac{u}{u_*} = \frac{1}{\kappa} \left( \ln \frac{z}{z_0} + \beta \frac{z}{L} \right)$$

$z_0$ : roughness height;  
 $\kappa$ : Von Kármán const;  
 $L$ : M-O scale.

$$L = -\frac{u_*^2}{\kappa \frac{g}{T} \rho c_p H_0}$$



The profile of a constant density flow past a flat plate for comparison:

$$\frac{u}{u_*} = \frac{1}{\kappa} \ln \left( \frac{9z u_*}{\nu} \right)$$

We can note that, the Reynolds number does not count in atmospheric flows.

## Summary

Most important atmospheric flow related physical phenomena beyond the scope of basic level fluid mechanics:

We have touched upon these topics

- Thermal stratification;
- Adiabatic compression and expansion due to vertical flows;
- Variation of density in vertical flows due to the hydrostatic pressure;
- Coriolis force;
- Turbulence in stratified medium;
- Moisture transport and phase changes;
- Surface energy balance involving the radiation heat transport, the heat storage and a number of other complex phenomena.

## Methodology

ANSYS FLUENT  
+ transformation system  
+ customized source terms

## CFD based atmospheric simulations

Gergely Kristóf Ph.D., Miklós Balogh, Norbert Rác  
29-th March 2009.

## Mathematical description

$$p = p_0 - \rho_0 \beta (T - T_0)$$

$$\nabla \cdot \mathbf{v} = 0$$

$$\frac{\partial}{\partial t} (\rho_0 \mathbf{v}) + \nabla \cdot (\rho_0 \mathbf{v} \otimes \mathbf{v}) = -\nabla p + \nabla \cdot \boldsymbol{\tau} + (p - \rho_0) \mathbf{g} + \mathbf{F}$$

$$\frac{\partial}{\partial t} (\rho_0 c_p T) + \nabla \cdot (\mathbf{v} \rho_0 c_p T) = \nabla \cdot (K_r \nabla T) + S_T$$

$$\frac{\partial}{\partial t} (\rho_0 k) + \nabla \cdot (\rho_0 \mathbf{v} k) = \nabla \cdot \left( \frac{\mu_t}{\sigma_k} \nabla k \right) + G_k + G_b - \rho_0 \epsilon + S_k$$

$$\frac{\partial}{\partial t} (\rho_0 \epsilon) + \nabla \cdot (\rho_0 \mathbf{v} \epsilon) = \nabla \cdot \left( \frac{\mu_t}{\sigma_\epsilon} \nabla \epsilon \right) + \rho_0 C_1 S \epsilon - \rho_0 C_2 \frac{\epsilon^2}{k + \sqrt{\nu \epsilon}} +$$

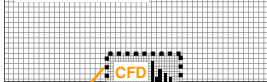
$$+ C_{1\epsilon} \frac{\epsilon}{k} C_{3\epsilon} G_b + S_\epsilon$$

Transformed variables  $\bar{p}, \bar{T}, \bar{\rho}, \mathbf{v}, z$

Customized volume sources

## Advantages of a CFD based model

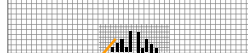
Meso scale model



model conversion interface

- Either the surface geometry is described in high details or (alternatively) meso-scale effects are taken into account.
- Bidirectional interface is a source of numerical errors eg. it can cause partial reflection.

CFD (with some changes)



grid refinement

- Higher accuracy
- No limits on geometrical precision
- Flexible meshing
- Advanced turbulent models
- Easy customization
- Advanced pre- and post processing tools

## Equilibrium profiles

up to the height of 11 km

$$\bar{T} = T_0 - \gamma z$$

$$\bar{p} = p_0 \left( \frac{T_0 - \gamma z}{T_0} \right)^{\frac{g}{R\gamma}}$$

$$\bar{\rho} = \rho_0 e^{-\zeta z}$$

$$T_0 = 288.15 \text{ K}$$

$$p_0 = 1.01325 \cdot 10^5 \text{ Pa}$$

$$\rho_0 = 1.225 \text{ kg/m}^3$$

$$\gamma = 0.65^\circ\text{C}/100\text{m}$$

$$g/(R\gamma) = 5.2553$$

$$\zeta = 10^{-4} \text{ m}^{-1}$$

Standard ISA profile

Approximate profile

Error bound is within 0.4% below 4000 m.

## Transformation expressions

$$T = \tilde{T} - T_0 + \bar{T}$$

$$p = \frac{\tilde{p}}{\rho_0} \cdot \bar{p} + \bar{p} = e^{-\zeta z} \cdot \bar{p} + \bar{p}$$

$$\rho = \bar{\rho} - \rho_0 + \bar{\rho}$$

$$z = -\frac{1}{\zeta} \text{Ln}(1 - \zeta z)$$

$$w = \frac{\tilde{w}}{\bar{p}} \bar{w} = \bar{w} e^{\zeta z}$$

## Two validation examples

Comparison with the results of water tank experiments

## Summary of source terms

In momentum equation:  $S_u = \rho_0 f v - \rho_0 \ell \bar{w} J$   
 $S_v = -\rho_0 f u$

$$S_w = \rho_0 (J^2 - 1) (\ell u J^{-1} + \beta (T - T_0) g) + \rho_0 \ell u J^{-1} + \zeta J (\bar{p} - \rho_0 \bar{w}^2)$$

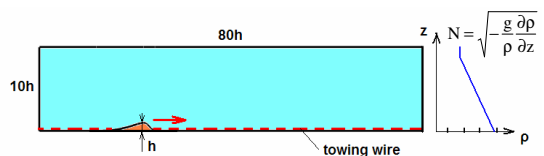
In the energy equation:  $S_T = J S_\Theta - \rho_0 c_p \bar{w} (\Gamma - \gamma) J$

In the transport equation of turbulent kinetic energy:  $S_k = -\beta g \frac{\mu_t}{Pr_t} (\Gamma - \gamma)$

In turbulent dissipation equation:  $S_\epsilon = -C_{1\epsilon} C_{3\epsilon} \frac{\epsilon}{k} \beta g \frac{\mu_t}{Pr_t} (\Gamma - \gamma)$   
 $\ell = 2 \Omega \cos \phi$   
 $f = 2 \Omega \sin \phi$   
 $J = (1 - \zeta z)^{-1}$

## Experimental setup

Gyüre, B. and Jánosi, I.M., 2003. Stratified flow over asymmetric and double bell-shaped obstacles. *Dynamics of Atmospheres and Oceans* 37, 155-170.



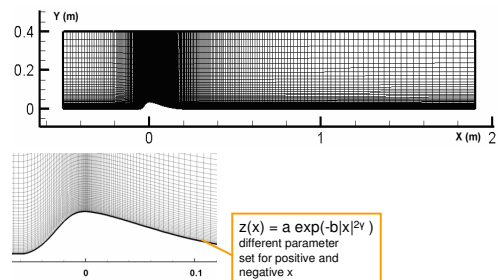
Uniformly stratified salt water:  
 $\rho = 1.03 - 1.00 \text{ g/cm}^3$   
 Typical towing speeds:  
 $U = 1-15 \text{ cm/s}$

Brunt-Vaisala frequency range:  
 $N = 1.09-1.55 \text{ 1/s}$   
 $Re_{exp} \approx 10^2-10^3$   
 Studied obstacle heights:  
 $h = 20\text{mm}, 40\text{mm}$

## Related publications

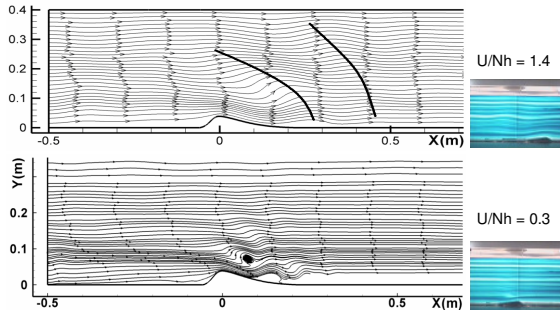
- [1] Kristóf G., Rácz N., Balogh M.: Adaptation of Pressure Based CFD Solvers for Mesoscale Atmospheric Problems. *Boundary-Layer Meteorol.*, 2008.
- [2] N. Rácz, G. Kristóf, T. Weidinger, M. Balogh: Simulation of gravity waves and model validation to laboratory experiments. *CD, Urban Air Quality Conf. Cyprus*, 2007.
- [3] G. Kristóf, N. Rácz, M. Balogh: Adaptation of pressure based CFD solvers to urban heat island convection problems. *CD, Urban Air Quality Conf. Cyprus*, 2007.
- [4] G. Kristóf, N. Rácz, Tamás Bányai, Norbert Rácz: Development of computational model for urban heat island convection using general purpose CFD solver. *ICUC6, 6-th Int. Conf. on Urban Climate*. Göteborg, pp. 822-825, 2006.
- [5] G. Kristóf, T. Weidinger, T. Bányai, N. Rácz, T. Gál, J. Unger: A városi hősziget által generált konvekció modellezése általános célú áramlási szoftverrel - példaként egy szegedi alkalmazással. *III. Magyar Földrajzi Konferencia, Budapest, 2006.*, Bp., CD
- [6] Kristóf G., Rácz N., Bányai T., Gál T., Unger J., Weidinger T.: A városi hősziget által generált konvekció modellezése általános célú áramlási szoftverrel - összehasonlítás kisminta kísérletekkel. *A 32. Meteorológiai Tudományos Napok előadásai. Országos Meteorológiai Szolgálat, Bp., 2006*
- [7] Dr. Lajos T., Dr. Kristóf G., Dr. Goricsán I., Rácz N.: Városklíma vizsgálatok a BME Áramlástan Tanszéken, hősziget numerikus szimulációja VÁHAVA projekt (A globális klímaváltozás: hazai hatások és válaszok) zárkonferenciája Bp. CD, 2006
- [8] Rácz N. és Kristóf G.: Hősziget cirkuláció kisminta méréseinek összehasonlítása saját fejlesztésű LES modellel. *Egyetemi Meteorológiai Füzetek No. 20 ELTE Meteorológiai Tanszék, Bp., 173-176, 2006.*
- [9] M. Balogh, G. Kristóf: Automated Grid Generation for Atmospheric Dispersion Simulations, pp. 1-6., *MICROCAD konferencia, Miskolc, 2007.*

## Numerical mesh



## Gravity waves

Gyüre, B. and Jánosi, I.M., 2003. Stratified flow over asymmetric and double bell-shaped obstacles. *Dynamics of Atmospheres and Oceans* 37, 155-170.



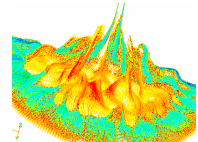
## Large eddy simulation in FLUENT

920 k prismatic cells

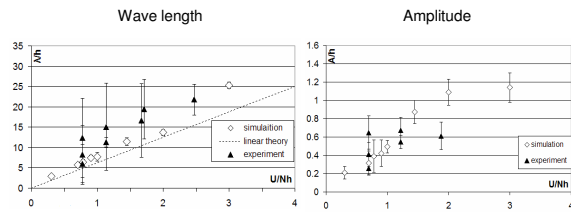


Streamlines colored by velocity magnitude

Fine structure of the thermal boundary layer



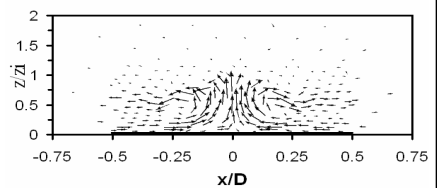
## Simulation vs. experimental data



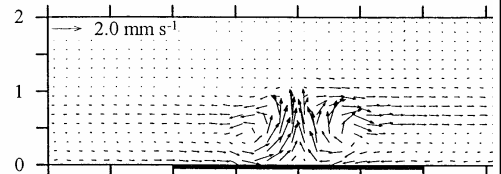
## Thermal convection

A.Cenedese, P.Monti: Interaction between an Inland Urban Heat Island and a Sea-Breeze Flow: A Laboratory Study, 2003.

CFD results

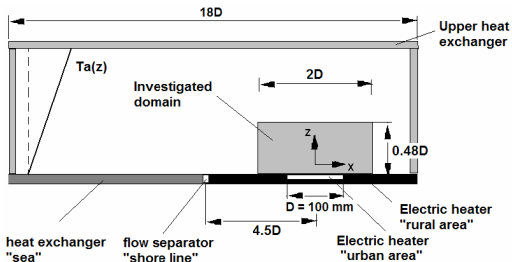


PIV results (Cenedese & Monti 2003)

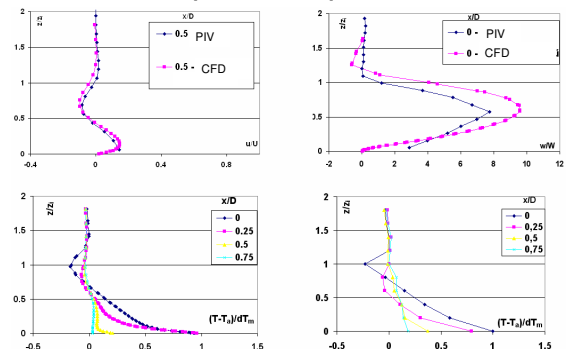


## Experimental setup

A.Cenedese, P.Monti: Interaction between an Inland Urban Heat Island and a Sea-Breeze Flow: A Laboratory Study, 2003.



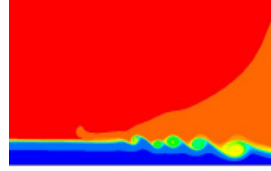
## Comparison of velocity and temperature profiles



## Some more application examples

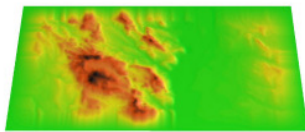
Full scale simulations

## Kelvin-Helmholtz instability

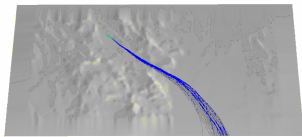


Com domain: 25 km x 5.5 km  
Temperature difference 20 °C

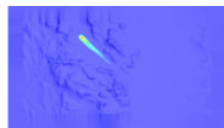
## Meso scale atmospheric dispersion



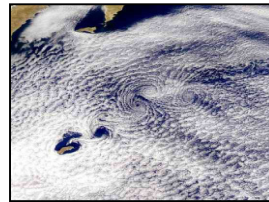
Orography of Pilis mountain



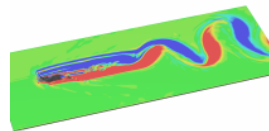
Evolution of surface concentration



## Von Kármán vortices behind a volcanic island

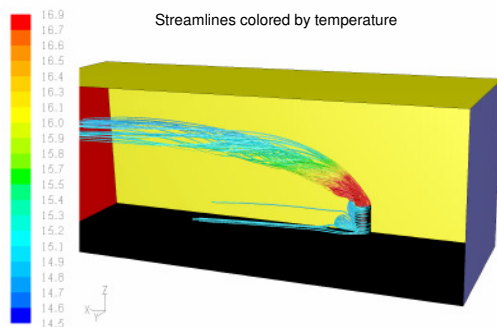


Satellite image about Guadalupe island



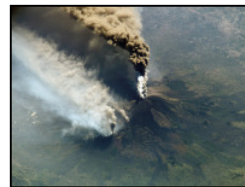
First CFD results

## Micro-scale atmospheric dispersion



Chimney height 180 m  
Standard (stable) temperature profile  
Wind speed: 3m/s  
Injection velocity: 5 m/s

## Targeted application areas



- Local circulation modeling:
  - Urban heat island convection, ventilation of cities;
  - Sea breeze;
  - Valley breeze.
- Power generation and pollution control:
  - Assessment of wind power potential, optimization of wind farms;
  - Plumes emitted by cooling towers and chimneys;
  - Dispersion of pollutant in the urban atmosphere.
- Research of meteorological phenomena:
  - Gravity waves;
  - Cloud formation;
  - Flow around high mountain.
- Simulation of disasters:
  - Large scale fires (e.g. in forest fires or town fires);
  - Volcanic plumes.