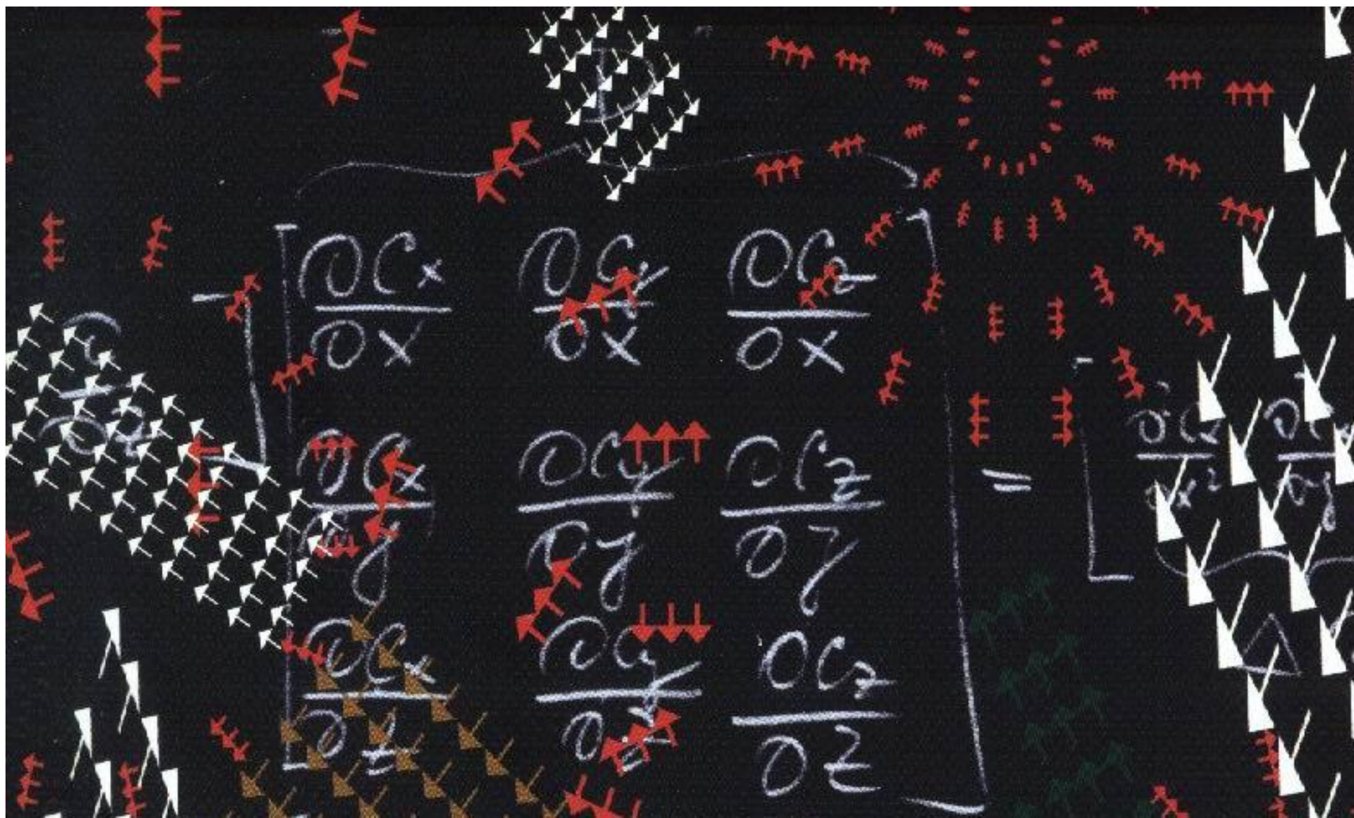
AM02 – Műszaki áramlástan I.
1.: Kis folyadékrész mozgása, kontinuitás

Dr. Szente Viktor
adjunktus
szente@ara.bme.hu

Budapesti Műszaki és
Gazdaságtudományi Egyetem
Gépészmérnöki Kar
Áramlástan Tanszék



ISMÉTLÉS: MÁTRIXMŰVELETEK

- Szorzás

$$\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} \cdot \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = a_1 \cdot b_1 + a_2 \cdot b_2 + a_3 \cdot b_3$$

$$\begin{bmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ z_1 & z_2 & z_3 \end{bmatrix} \cdot \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} x_1 \cdot a_1 + x_2 \cdot a_2 + x_3 \cdot a_3 \\ y_1 \cdot a_1 + y_2 \cdot a_2 + y_3 \cdot a_3 \\ z_1 \cdot a_1 + z_2 \cdot a_2 + z_3 \cdot a_3 \end{bmatrix}$$

- Keresztszorzás

$$\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} \times \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} a_2 \cdot b_3 - a_3 \cdot b_2 \\ a_3 \cdot b_1 - a_1 \cdot b_3 \\ a_1 \cdot b_2 - a_2 \cdot b_1 \end{bmatrix}$$

- Transzponálás: főátlóra tükrözés

Az áramlástanban alkalmazott fizikai mennyiségek jellemzésére

Skalártér (pl. nyomás) esetén a változás az egyik dimenzió mentén leírható:

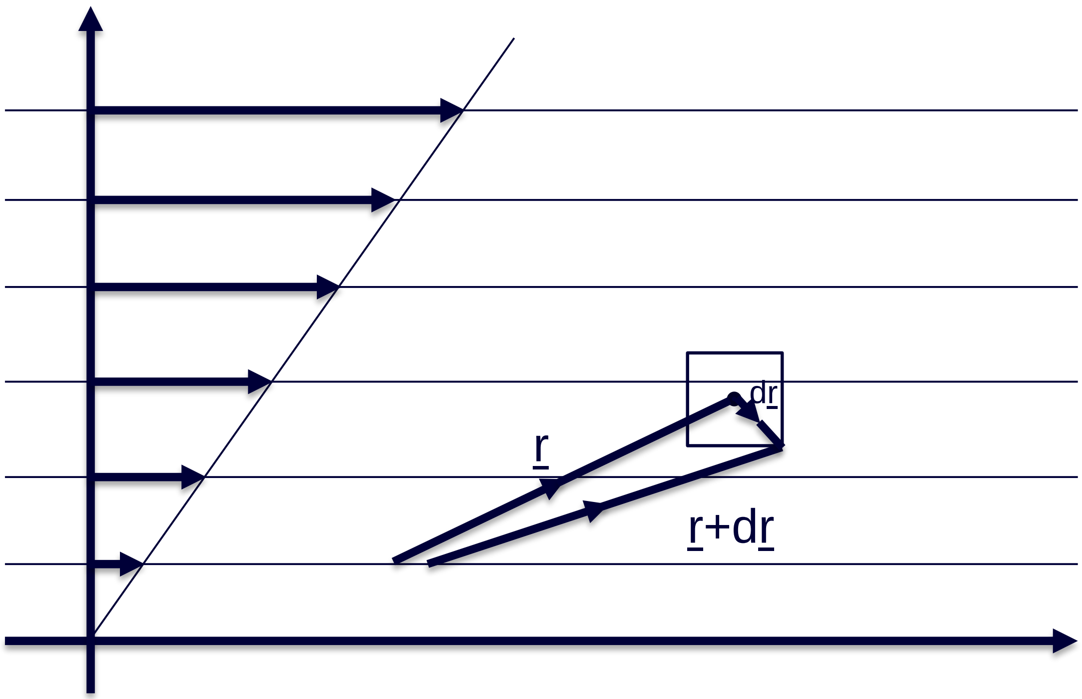
$$p(x+dx) \cong p(x) + \frac{\partial p}{\partial x} \cdot dx \quad \text{(lineáris közelítés)}$$

Mindhárom dimenzió mentén: gradiens:

$$p(\underline{r}+d\underline{r}) \cong p(\underline{r}) + \underline{grad}p \cdot d\underline{r} = p(\underline{r}) + \frac{\partial p}{\partial x} \cdot dx + \frac{\partial p}{\partial y} \cdot dy + \frac{\partial p}{\partial z} \cdot dz$$

KIS FOLYADÉKRÉSZ MOZGÁSA

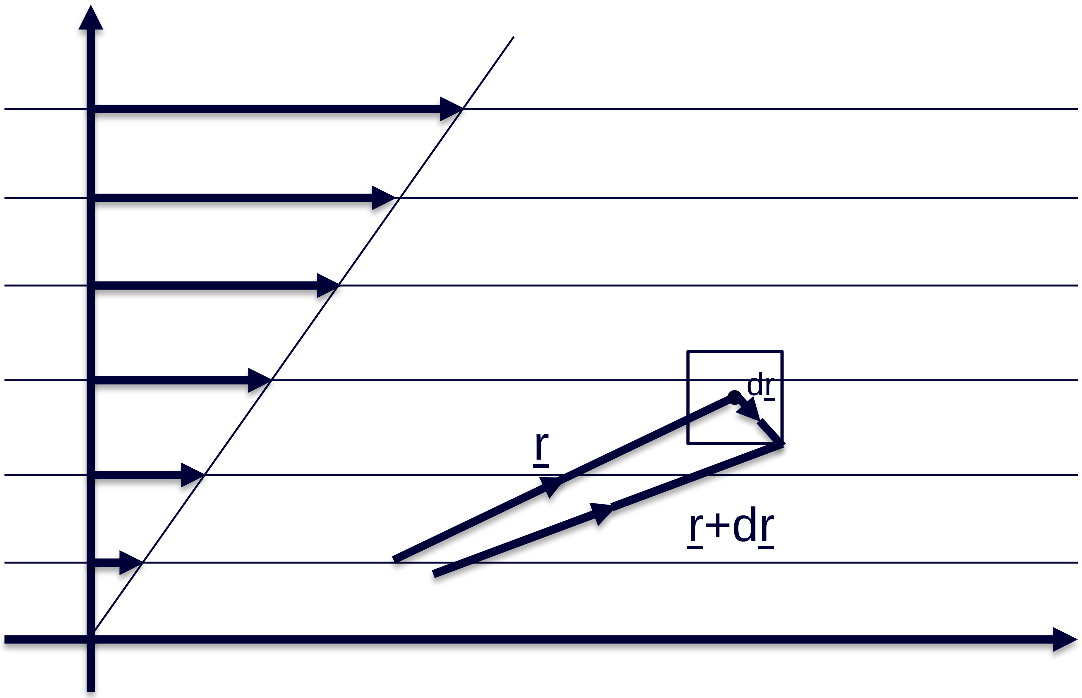
Stacioner áramlás



Hogyan mozog az elemi méretű folyadékhasáb?
Megtartja eredeti helyzetét?
Megtartja eredeti alakját?

Ehhez tudnunk kell hogy az élei hogy mozdulnak el a szimmetriatengelyéhez képest.

KIS FOLYADÉKRÉSZ MOZGÁSA

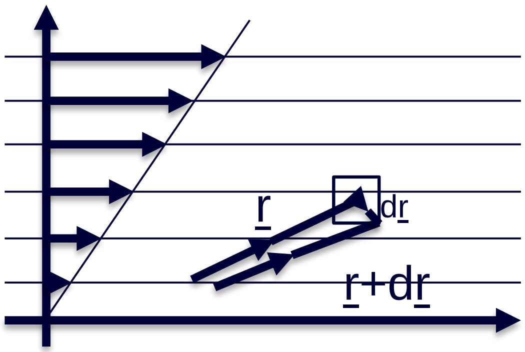


$$\underline{v}(\underline{r}+d\underline{r}) \cong \underline{v}(\underline{r}) + \underline{\underline{D}} \cdot d\underline{r}$$

$$\underline{\underline{D}} = \begin{bmatrix} \frac{\partial v_x}{\partial x} & \frac{\partial v_x}{\partial y} & \frac{\partial v_x}{\partial z} \\ \frac{\partial v_y}{\partial x} & \frac{\partial v_y}{\partial y} & \frac{\partial v_y}{\partial z} \\ \frac{\partial v_z}{\partial x} & \frac{\partial v_z}{\partial y} & \frac{\partial v_z}{\partial z} \end{bmatrix}$$

Deriválttenzor: a sebességtér hely szerinti változása

KIS FOLYADÉKRÉSZ MOZGÁSA



Deriválttenzor felbontása:

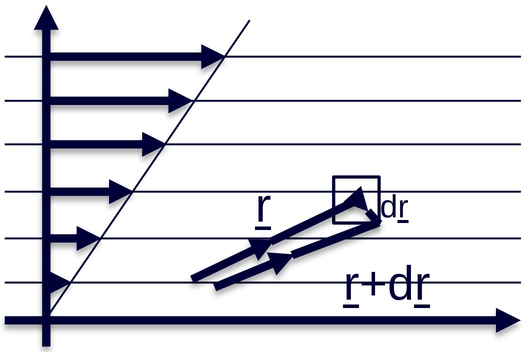
$$\underline{\underline{D}} = \underbrace{\frac{1}{2}\left(\underline{\underline{D}} + \underline{\underline{D}}^T\right)}_{\text{1. tag}} + \underbrace{\frac{1}{2}\left(\underline{\underline{D}} - \underline{\underline{D}}^T\right)}_{\text{2. tag}}$$

$$\underline{\underline{D}} = \begin{bmatrix} \frac{\partial v_x}{\partial x} & \frac{\partial v_x}{\partial y} & \frac{\partial v_x}{\partial z} \\ \frac{\partial v_y}{\partial x} & \frac{\partial v_y}{\partial y} & \frac{\partial v_y}{\partial z} \\ \frac{\partial v_z}{\partial x} & \frac{\partial v_z}{\partial y} & \frac{\partial v_z}{\partial z} \end{bmatrix} \qquad \underline{\underline{D}}^T = \begin{bmatrix} \frac{\partial v_x}{\partial x} & \frac{\partial v_y}{\partial x} & \frac{\partial v_z}{\partial x} \\ \frac{\partial v_x}{\partial y} & \frac{\partial v_y}{\partial y} & \frac{\partial v_z}{\partial y} \\ \frac{\partial v_x}{\partial z} & \frac{\partial v_y}{\partial z} & \frac{\partial v_z}{\partial z} \end{bmatrix}$$

1. tag:

$$\underline{\underline{A}}_s = \frac{1}{2} \cdot \begin{bmatrix} \frac{\partial v_x}{\partial x} + \frac{\partial v_x}{\partial x} & \frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x} & \frac{\partial v_x}{\partial z} + \frac{\partial v_z}{\partial x} \\ \frac{\partial v_y}{\partial x} + \frac{\partial v_x}{\partial y} & \frac{\partial v_y}{\partial y} + \frac{\partial v_y}{\partial y} & \frac{\partial v_y}{\partial z} + \frac{\partial v_z}{\partial y} \\ \frac{\partial v_z}{\partial x} + \frac{\partial v_x}{\partial z} & \frac{\partial v_z}{\partial y} + \frac{\partial v_y}{\partial z} & \frac{\partial v_z}{\partial z} + \frac{\partial v_z}{\partial z} \end{bmatrix}$$

szimmetrikus



Deriválttenzor felbontása:

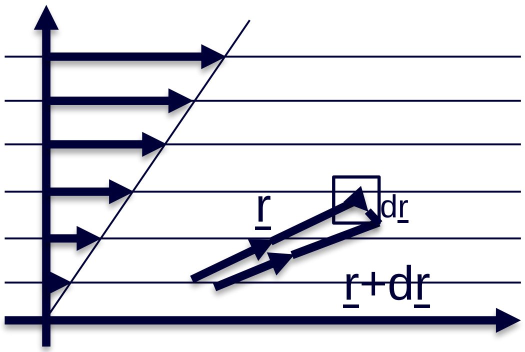
$$\underline{\underline{D}} = \underbrace{\frac{1}{2}\left(\underline{\underline{D}} + \underline{\underline{D}}^T\right)}_{\text{1. tag}} + \underbrace{\frac{1}{2}\left(\underline{\underline{D}} - \underline{\underline{D}}^T\right)}_{\text{2. tag}}$$

$$\underline{\underline{D}} = \begin{bmatrix} \frac{\partial v_x}{\partial x} & \frac{\partial v_x}{\partial y} & \frac{\partial v_x}{\partial z} \\ \frac{\partial v_y}{\partial x} & \frac{\partial v_y}{\partial y} & \frac{\partial v_y}{\partial z} \\ \frac{\partial v_z}{\partial x} & \frac{\partial v_z}{\partial y} & \frac{\partial v_z}{\partial z} \end{bmatrix} \qquad \underline{\underline{D}}^T = \begin{bmatrix} \frac{\partial v_x}{\partial x} & \frac{\partial v_y}{\partial x} & \frac{\partial v_z}{\partial x} \\ \frac{\partial v_x}{\partial y} & \frac{\partial v_y}{\partial y} & \frac{\partial v_z}{\partial y} \\ \frac{\partial v_x}{\partial z} & \frac{\partial v_y}{\partial z} & \frac{\partial v_z}{\partial z} \end{bmatrix}$$

2. tag:

$$\underline{\underline{A}}_\Omega = \frac{1}{2} \cdot \begin{bmatrix} \frac{\partial v_x}{\partial x} - \frac{\partial v_x}{\partial x} & \frac{\partial v_x}{\partial y} - \frac{\partial v_y}{\partial x} & \frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x} \\ \frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y} & \frac{\partial v_y}{\partial y} - \frac{\partial v_y}{\partial y} & \frac{\partial v_y}{\partial z} - \frac{\partial v_z}{\partial y} \\ \frac{\partial v_z}{\partial x} - \frac{\partial v_x}{\partial z} & \frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z} & \frac{\partial v_z}{\partial z} - \frac{\partial v_z}{\partial z} \end{bmatrix}$$

KIS FOLYADÉKRÉSZ MOZGÁSA



Deriválttenzor felbontása:

$$\underline{\underline{D}} = \underbrace{\frac{1}{2}\left(\underline{\underline{D}} + \underline{\underline{D}}^T\right)}_{\text{1. tag}} + \underbrace{\frac{1}{2}\left(\underline{\underline{D}} - \underline{\underline{D}}^T\right)}_{\text{2. tag}}$$

$$\underline{\underline{D}} = \begin{bmatrix} \frac{\partial v_x}{\partial x} & \frac{\partial v_x}{\partial y} & \frac{\partial v_x}{\partial z} \\ \frac{\partial v_y}{\partial x} & \frac{\partial v_y}{\partial y} & \frac{\partial v_y}{\partial z} \\ \frac{\partial v_z}{\partial x} & \frac{\partial v_z}{\partial y} & \frac{\partial v_z}{\partial z} \end{bmatrix} \qquad \underline{\underline{D}}^T = \begin{bmatrix} \frac{\partial v_x}{\partial x} & \frac{\partial v_y}{\partial x} & \frac{\partial v_z}{\partial x} \\ \frac{\partial v_x}{\partial y} & \frac{\partial v_y}{\partial y} & \frac{\partial v_z}{\partial y} \\ \frac{\partial v_x}{\partial z} & \frac{\partial v_y}{\partial z} & \frac{\partial v_z}{\partial z} \end{bmatrix}$$

2. tag: $$\underline{\underline{A}}_\Omega = \frac{1}{2}\cdot \begin{bmatrix} 0 & \frac{\partial v_x}{\partial y} - \frac{\partial v_y}{\partial x} & \frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x} \\ \frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y} & 0 & \frac{\partial v_y}{\partial z} - \frac{\partial v_z}{\partial y} \\ \frac{\partial v_z}{\partial x} - \frac{\partial v_x}{\partial z} & \frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z} & 0 \end{bmatrix}$$ antiszimmetrikus

ISMÉTLÉS: ROTÁCIÓ

$$\underline{rot}\ \underline{v} = \underline{\nabla} \times \underline{v} = \begin{bmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{bmatrix} \times \begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix} = \begin{bmatrix} \frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z} \\ \frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x} \\ \frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y} \end{bmatrix} = 2 \cdot \underline{\Omega}$$

ISMÉTLÉS: ROTÁCIÓ

$$\underline{rot}\ \underline{v} = \underline{\nabla} \times \underline{v} = \begin{bmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{bmatrix} \times \begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix} = \begin{bmatrix} \frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z} \\ \frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x} \\ \frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y} \end{bmatrix} = 2 \cdot \underline{\Omega}$$

ez alapján:

$$\underline{\underline{A}}_{\Omega} = \frac{1}{2} \cdot \begin{bmatrix} 0 & \frac{\partial v_x}{\partial y} - \frac{\partial v_y}{\partial x} & \frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x} \\ \frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y} & 0 & \frac{\partial v_y}{\partial z} - \frac{\partial v_z}{\partial y} \\ \frac{\partial v_z}{\partial x} - \frac{\partial v_x}{\partial z} & \frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z} & 0 \end{bmatrix} = \frac{1}{2} \cdot \begin{bmatrix} 0 & -(\underline{rot}\ \underline{v})_z & (\underline{rot}\ \underline{v})_y \\ (\underline{rot}\ \underline{v})_z & 0 & -(\underline{rot}\ \underline{v})_x \\ -(\underline{rot}\ \underline{v})_y & (\underline{rot}\ \underline{v})_x & 0 \end{bmatrix}$$

Vagyis az antiszimmetrikus mátrix elemei a $\underline{rot}\ \underline{v}$ vektor komponensei.

$$\underline{\underline{A}}_\Omega = \frac{1}{2}\cdot\begin{bmatrix} 0 & \frac{\partial v_x}{\partial y}-\frac{\partial v_y}{\partial x} & \frac{\partial v_x}{\partial z}-\frac{\partial v_z}{\partial x} \\ \frac{\partial v_y}{\partial x}-\frac{\partial v_x}{\partial y} & 0 & \frac{\partial v_y}{\partial z}-\frac{\partial v_z}{\partial y} \\ \frac{\partial v_z}{\partial x}-\frac{\partial v_x}{\partial z} & \frac{\partial v_z}{\partial y}-\frac{\partial v_y}{\partial z} & 0 \end{bmatrix} \qquad d\underline{r} = \begin{bmatrix} dx \\ dy \\ dz \end{bmatrix}$$

$$\underline{\underline{A}}_\Omega \cdot d\underline{r} = \frac{1}{2}\cdot\begin{bmatrix} \cancel{0\cdot dx} + \left(\frac{\partial v_x}{\partial y}-\frac{\partial v_y}{\partial x}\right)\cdot dy + \left(\frac{\partial v_x}{\partial z}-\frac{\partial v_z}{\partial x}\right)\cdot dz \\ \left(\frac{\partial v_y}{\partial x}-\frac{\partial v_x}{\partial y}\right)\cdot dx + \cancel{0\cdot dy} + \left(\frac{\partial v_y}{\partial z}-\frac{\partial v_z}{\partial y}\right)\cdot dz \\ \left(\frac{\partial v_z}{\partial x}-\frac{\partial v_x}{\partial z}\right)\cdot dx + \left(\frac{\partial v_z}{\partial y}-\frac{\partial v_y}{\partial z}\right)\cdot dy + \cancel{0\cdot dz} \end{bmatrix} =$$

$$= \frac{1}{2}\cdot\begin{bmatrix} \left(\frac{\partial v_x}{\partial y}-\frac{\partial v_y}{\partial x}\right)\cdot dy + \left(\frac{\partial v_x}{\partial z}-\frac{\partial v_z}{\partial x}\right)\cdot dz \\ \left(\frac{\partial v_y}{\partial x}-\frac{\partial v_x}{\partial y}\right)\cdot dx + \left(\frac{\partial v_y}{\partial z}-\frac{\partial v_z}{\partial y}\right)\cdot dz \\ \left(\frac{\partial v_z}{\partial x}-\frac{\partial v_x}{\partial z}\right)\cdot dx + \left(\frac{\partial v_z}{\partial y}-\frac{\partial v_y}{\partial z}\right)\cdot dy \end{bmatrix}$$

ISMÉTLÉS: ROTÁCIÓ

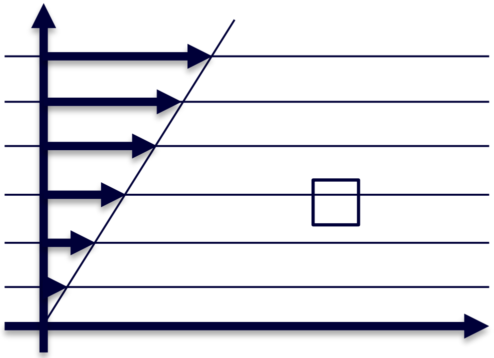
$$\underline{rot}\ \underline{v} = \begin{bmatrix} \frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z} \\ \frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x} \\ \frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y} \end{bmatrix} \quad d\underline{r} = \begin{bmatrix} dx \\ dy \\ dz \end{bmatrix} \quad \underline{\underline{A}}_\Omega \cdot d\underline{r} = \frac{1}{2} \cdot \begin{bmatrix} \left(\frac{\partial v_x}{\partial y} - \frac{\partial v_y}{\partial x}\right) \cdot dy + \left(\frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x}\right) \cdot dz \\ \left(\frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y}\right) \cdot dx + \left(\frac{\partial v_y}{\partial z} - \frac{\partial v_z}{\partial y}\right) \cdot dz \\ \left(\frac{\partial v_z}{\partial x} - \frac{\partial v_x}{\partial z}\right) \cdot dx + \left(\frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z}\right) \cdot dy \end{bmatrix}$$

$$\underline{rot}\ \underline{v} \times d\underline{r} = \begin{bmatrix} \left(\frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x}\right) dz - \left(\frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y}\right) dy \\ \left(\frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y}\right) dx - \left(\frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z}\right) dz \\ \left(\frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z}\right) dy - \left(\frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x}\right) dx \end{bmatrix} = \begin{bmatrix} \left(\frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x}\right) dz + \left(\frac{\partial v_x}{\partial y} - \frac{\partial v_y}{\partial x}\right) dy \\ \left(\frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y}\right) dx + \left(\frac{\partial v_y}{\partial z} - \frac{\partial v_z}{\partial y}\right) dz \\ \left(\frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z}\right) dy + \left(\frac{\partial v_z}{\partial x} - \frac{\partial v_x}{\partial z}\right) dx \end{bmatrix}$$

$$\underline{\underline{A}}_\Omega \cdot d\underline{r} = \frac{1}{2} \cdot \underline{rot}\ \underline{v} \times d\underline{r} = \underline{\Omega} \times d\underline{r}$$

A sebességtér örvényessége és a folyadékrészek $\underline{\Omega}$ forgási szögsebessége között szoros kapcsolat van.

SEBESSÉGVÁLTOZÁS A GYAKORLATBAN

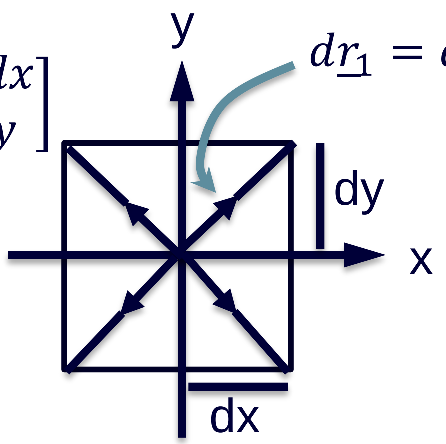


Tegyük fel, hogy a sebességtér: $\underline{v} = 4y \cdot \underline{i}$ vagyis $\underline{v} = v_x(y)$

Elemi hasáb:

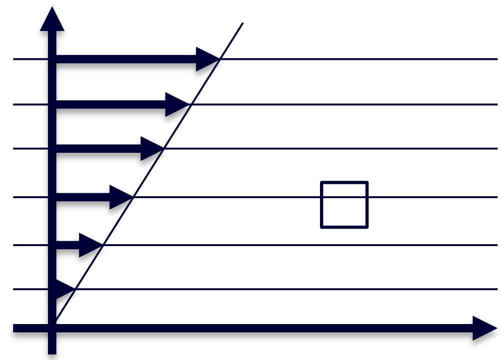
$$d\underline{r}_2 = \begin{bmatrix} -dx \\ dy \end{bmatrix}$$

$$d\underline{r}_1 = dx \cdot \underline{i} + dy \cdot \underline{j} = \begin{bmatrix} dx \\ dy \end{bmatrix}$$



Fejtsük ki:

$$\underline{\underline{D}} = \begin{bmatrix} \frac{\partial v_x}{\partial x} & \frac{\partial v_x}{\partial y} & \frac{\partial v_x}{\partial z} \\ \frac{\partial v_y}{\partial x} & \frac{\partial v_y}{\partial y} & \frac{\partial v_y}{\partial z} \\ \frac{\partial v_z}{\partial x} & \frac{\partial v_z}{\partial y} & \frac{\partial v_z}{\partial z} \end{bmatrix} = \begin{bmatrix} 0 & 4 \\ 0 & 0 \end{bmatrix}$$



A sebességtér: $\underline{v} = 4y \cdot \underline{i}$

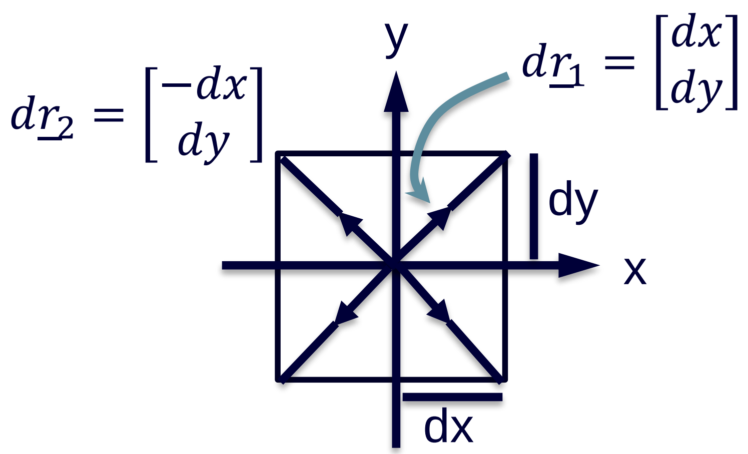
$$\underline{\underline{A}}_s = \frac{1}{2} \cdot \begin{bmatrix} \frac{\partial v_x}{\partial x} + \frac{\partial v_x}{\partial x} & \frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x} & \frac{\partial v_x}{\partial z} + \frac{\partial v_z}{\partial x} \\ \frac{\partial v_y}{\partial x} + \frac{\partial v_x}{\partial y} & \frac{\partial v_y}{\partial y} + \frac{\partial v_y}{\partial y} & \frac{\partial v_y}{\partial z} + \frac{\partial v_z}{\partial y} \\ \frac{\partial v_z}{\partial x} + \frac{\partial v_x}{\partial z} & \frac{\partial v_z}{\partial y} + \frac{\partial v_y}{\partial z} & \frac{\partial v_z}{\partial z} + \frac{\partial v_z}{\partial z} \end{bmatrix} = \begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix}$$

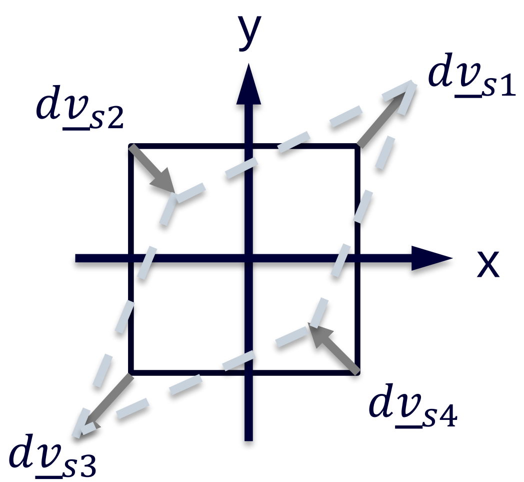
$$\underline{\underline{A}}_\Omega = \frac{1}{2} \cdot \begin{bmatrix} 0 & \frac{\partial v_x}{\partial y} - \frac{\partial v_y}{\partial x} & \frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x} \\ \frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y} & 0 & \frac{\partial v_y}{\partial z} - \frac{\partial v_z}{\partial y} \\ \frac{\partial v_z}{\partial x} - \frac{\partial v_x}{\partial z} & \frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z} & 0 \end{bmatrix} = \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$$

SEBESSÉGVÁLTOZÁS A GYAKORLATBAN

Deformáció:

A sebességtér: $\underline{v} = 4y \cdot \underline{i}$





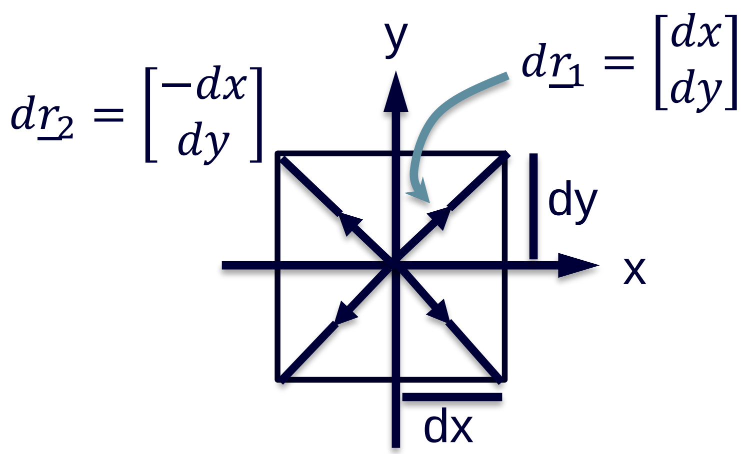
$$\underline{\underline{A}}_s = \begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix}$$

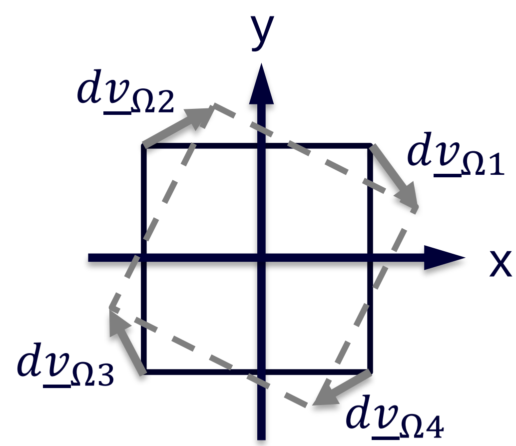
$$d\underline{v}_{s1} = \underline{\underline{A}}_s \cdot d\underline{r}_1 = \begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix} \cdot \begin{bmatrix} dx \\ dy \end{bmatrix} = \begin{bmatrix} 2 \cdot dy \\ 2 \cdot dx \end{bmatrix}$$

$$d\underline{v}_{s2} = \underline{\underline{A}}_s \cdot d\underline{r}_2 = \begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix} \cdot \begin{bmatrix} -dx \\ dy \end{bmatrix} = \begin{bmatrix} 2 \cdot dy \\ -2 \cdot dx \end{bmatrix}$$

Rotáció:

A sebességtér: $\underline{v} = 4y \cdot \underline{i}$



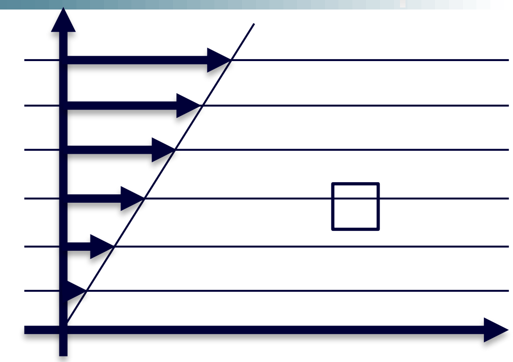


$$\underline{\underline{A}}_{\Omega} = \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$$

$$d\underline{v}_{\Omega 1} = \underline{\underline{A}}_{\Omega} \cdot d\underline{r}_1 = \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix} \cdot \begin{bmatrix} dx \\ dy \end{bmatrix} = \begin{bmatrix} 2 \cdot dy \\ -2 \cdot dx \end{bmatrix}$$

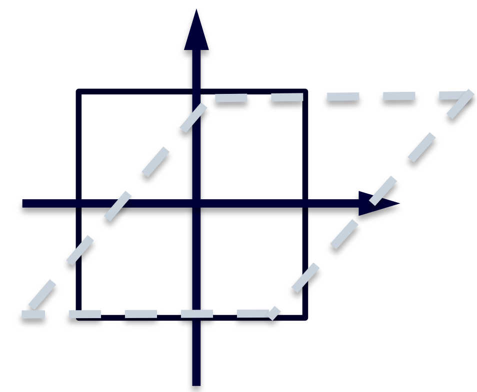
$$d\underline{v}_{\Omega 2} = \underline{\underline{A}}_{\Omega} \cdot d\underline{r}_2 = \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix} \cdot \begin{bmatrix} -dx \\ dy \end{bmatrix} = \begin{bmatrix} 2 \cdot dy \\ 2 \cdot dx \end{bmatrix}$$

A TELJES SEBESSÉGVÁLTOZÁS



$$\underline{v}(\underline{r}+d\underline{r}) \cong \underline{v}(\underline{r}) + \underline{\underline{D}} \cdot d\underline{r} = \underline{v}(\underline{r}) + \underline{\underline{A}}_s \cdot d\underline{r} + \underline{\underline{A}}_\Omega \cdot d\underline{r}$$

Párhuzamos elmozdulás
Deformálódás
Elfordulás



Ennek megfelelően:

$\underline{\underline{A}}_s$ Alakváltozási sebesség tenzor

$\underline{\underline{A}}_\Omega$ Örvénytenzor

Ha $\underline{\underline{D}}$ szimmetrikus, akkor $\underline{\underline{A}}_\Omega = 0$, vagyis $\underline{rot}\ \underline{v} = 0$, ekkor létezik a sebességtérnek potenciálja (Ψ), amire igaz, hogy $\underline{v} = \underline{grad}\Psi$

KONTINUITÁS

Zárt felületen a többletkiáramlás: $q_v = \int_A \underline{v}\, d\underline{A}$ $q_m = \int_A \rho\, \underline{v}\, d\underline{A}$

Többletkiáramlás: a felületen belül (= a térfogatban) a sűrűség változik.

Ez a változás: $\int_V \frac{\partial \rho}{\partial t} dV$

Ha a tömegáram pozitív, akkor több áramlik ki mint be: a térfogatban fogy a tömeg,

a sűrűség csökken: $\int_A \rho\, \underline{v}\, d\underline{A} = -\int_V \frac{\partial \rho}{\partial t} dV$

Gauss-Osztrogradszkij tétel alapján: $\int_A \rho\, \underline{v}\, d\underline{A} = \int_V div\, \rho\underline{v}\, dV = -\int_V \frac{\partial \rho}{\partial t} dV$

Innen: $\int_V div\, \rho\underline{v}\, dV + \int_V \frac{\partial \rho}{\partial t} dV = 0 = \int_V \left(div\, \rho\underline{v} + \frac{\partial \rho}{\partial t} \right) dV$

KONTINUITÁS

$$\int_V \left(div\ \rho \underline{v} + \frac{\partial \rho}{\partial t} \right) dV = 0$$

Ez akkor igaz, ha: $div\ \rho \underline{v} + \frac{\partial \rho}{\partial t} = 0$

A divergencia a deriválási szabályok szerint felbontva: $div\ \rho \underline{v} = \underline{v}\ \underline{grad}\ \rho + \rho\ div \underline{v}$

Átrendezve: $\frac{\partial \rho}{\partial t} + \underline{v}\ \underline{grad}\ \rho = -\rho\ div \underline{v}$

Lokális változás, az idő függvényében
Konvektív változás, azért változik mert A-ból B-be jut

Ha A-ból B-be eljut, akkor ρ_A-nak addigra ρ_B-vé kell változnia.

A változás értéke: $\underline{grad} \rho \cdot d\underline{s}$ ahol $d\underline{s} = \underline{v} \cdot dt$ ezért a változás: $\underline{grad} \rho \cdot \underline{v} \cdot dt$

Mivel egységnyi idő alatti változást vizsgáljuk, ezért marad a: $\underline{v}\ \underline{grad}\ \rho$

KONTINUITÁS

Az előzőeknek megfelelően a sűrűség teljes változása: $\frac{d\rho}{dt} = \frac{\partial \rho}{\partial t} + \underline{v} \cdot \underline{grad}\rho$

Ugyanez sebességtérre: $\underline{v} = \underline{v}(\underline{r}, t)$ $\quad \frac{d\underline{v}}{dt} = ?$

X irányú sebességkomponensre:

$$\frac{dv_x}{dt} = \frac{\partial v_x}{\partial t} + \underline{v} \cdot \underline{grad}\, v_x = \frac{\partial v_x}{\partial t} + v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y} + v_z \frac{\partial v_x}{\partial z}$$

$$\frac{d\underline{v}}{dt} = \begin{bmatrix} \frac{\partial v_x}{\partial t} + v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y} + v_z \frac{\partial v_x}{\partial z} \\ \frac{\partial v_y}{\partial t} + v_x \frac{\partial v_y}{\partial x} + v_y \frac{\partial v_y}{\partial y} + v_z \frac{\partial v_y}{\partial z} \\ \frac{\partial v_z}{\partial t} + v_x \frac{\partial v_z}{\partial x} + v_y \frac{\partial v_z}{\partial y} + v_z \frac{\partial v_z}{\partial z} \end{bmatrix} = \frac{\partial \underline{v}}{\partial t} + \underline{\underline{D}}\, \underline{v}$$

Konvektív gyorsulás

Lokális gyorsulás

Példa: konfúzorban a stacioner áramlás sebessége 4 m/s-ról 6 m/s-ra nő egy 0.5 m-es szakaszon.

Gyorsulás: csak konvektív, csak X irányú komponens.

$$\underline{\underline{D}}\,\underline{v} = v_x \frac{\partial v_x}{\partial x}$$

$$\frac{\partial v_x}{\partial x} = \frac{6-4}{0.5} = 4\frac{1}{s}$$ Közelítés!

$$v_x = \frac{4+6}{2} = 5\frac{m}{s}$$ Ez is!

Eredmény: $a_{konv} = 5\frac{m}{s} \cdot 4\frac{1}{s} = 20\frac{m}{s^2}$