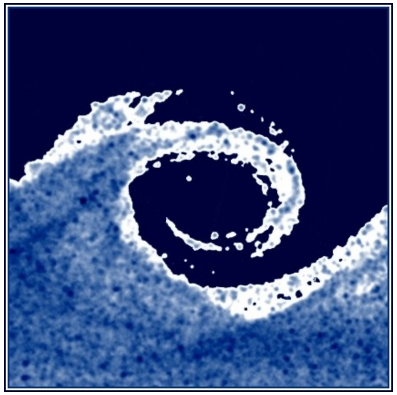

Post-Processing of Large-Eddy Simulation data: Coherent Structure concept

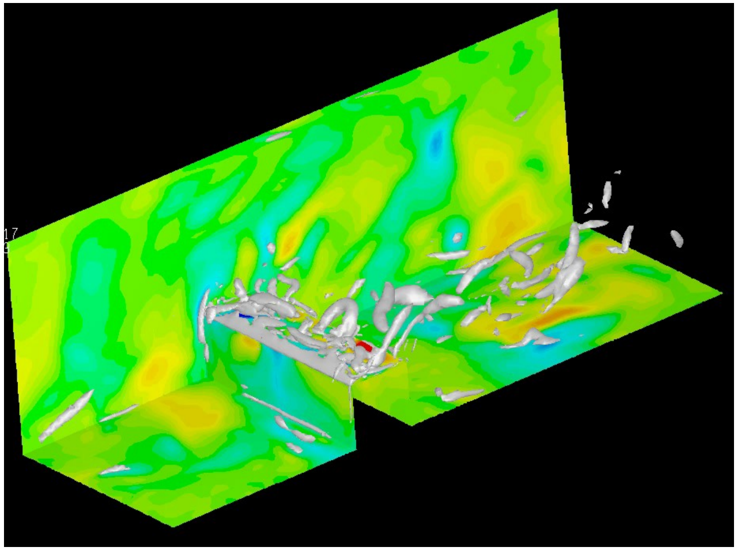

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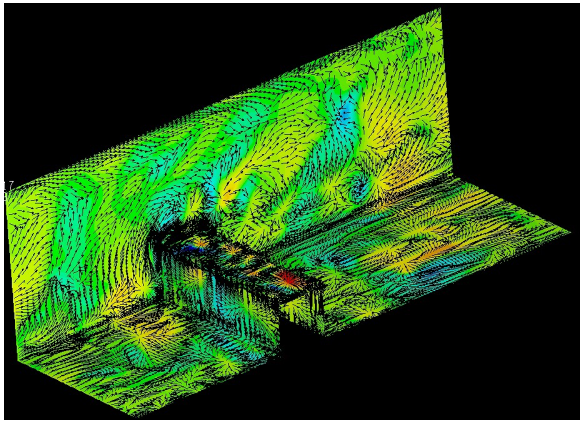
Table of contents

Motivation

Huge amount of data from LES!

Example:
- Wall shear stress vector
- Pressure fluctuation



How to understand the flowfield,
how to find the sources of sound?

➔ Find structures in the flow to facilitate its understanding!
➔ Build a skeleton (model) of the flow
➔ Connection between structures and emitted sound.

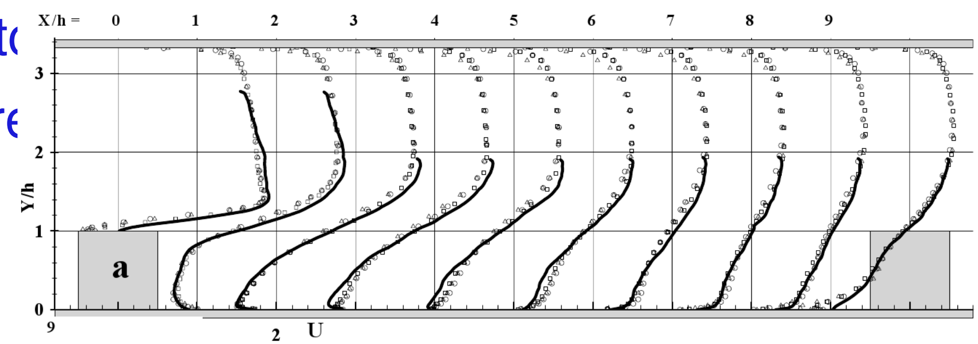
Motivation

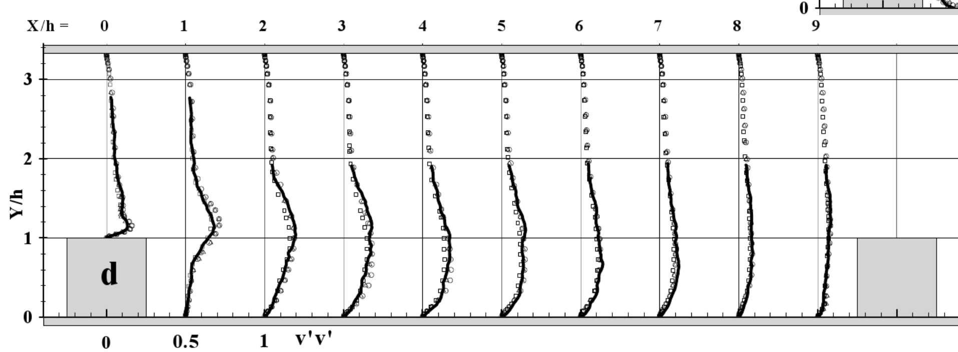
Traditional view of turbulence:

- turbulent motion is completly random and can be described by statistical means

- typical statistical parameters:
  - mean, rms, skewness, et
  - spatial and temporal corre scales)

Mean streamwise velocity





Wall normal velocity rms

(Lohász2006)

Motivation

- To understand their behaviour transport equations were developed and the terms of the equations were analysed
  - example1: turbulent kinetic energy transport equation (production, diffusion, dissipation can be identified)

$$\frac{\bar{D}k}{\bar{D}t} + \nabla \cdot \boldsymbol{T}' = \mathcal{P} - \varepsilon$$

$$\frac{\bar{D}}{\bar{D}t}\langle u_i u_j \rangle + \frac{\partial}{\partial x_k}\left(T_{kij}^{(\nu)} + T_{kij}^{(p')} + T_{kij}^{(u)} \right) = \mathcal{P}_{ij} + \mathcal{R}_{ij}^{(a)} - \varepsilon_{ij}$$

  - example2: Reynolds stress transport equation (interaction between the different components can be also identified)

(Pope2000)

Coherent structure concept

Coherent structure (CS) concept:

Turbulent motion can be decomposed into three parts

Reynolds decomposition

$$\varphi = \bar{\varphi} + \tilde{\varphi}$$

Triple decomposition

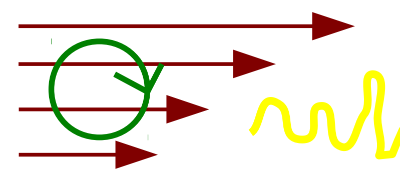
$$\varphi = \bar{\varphi} + \tilde{\varphi}_c + \tilde{\varphi}_b$$

$\bar{\varphi}$ Average

$\tilde{\varphi} = \tilde{\varphi}_c + \tilde{\varphi}_b$ Fluctuation

$\tilde{\varphi}_c$ Coherent motion

$\tilde{\varphi}_b$ Turbulent background



An important part of the fluctuation can be characterised by the motion of regular fluid structure so called coherent structures

Coherent structure concept

The two mostly cited definitions:

Hussain1986:

„A coherent structure is a connected turbulent fluid mass with instantaneously phase correlated vorticity over its spatial extent."

Robinson1991:

„... a three-dimensional region of flow over which at least one fundamental flow variable (velocity component, temperature, etc.) exhibits significant correlation with itself or with another variable over a range of space and/or time that is significantly larger than the smallest local scale of the flow ..."

Coherent structure concept

Structure eduction:

- Select a fundamental variable, (which is related to vorticity for *Hussain1986*) which will identify the *structure*.
- Check the *coherence*, by carrying out phase averaging.

Simplified definition:
Coherent structure = typical vortex

Vortex detection criteria (the fundamental variable)

What is a vortex?

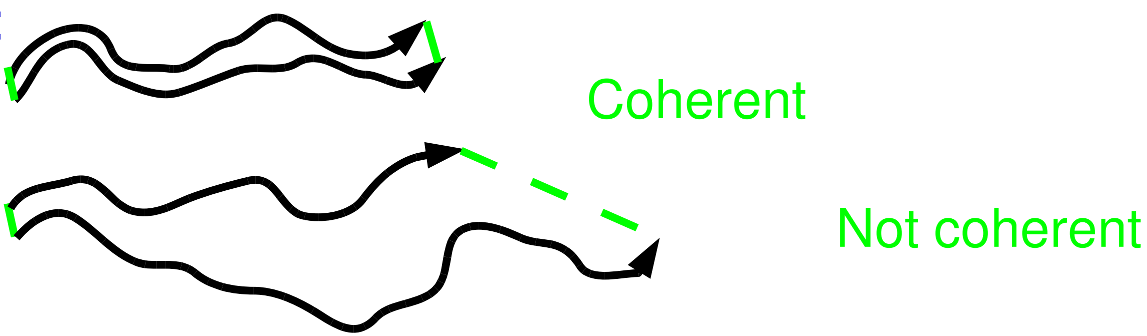
- Something rotating
- Examples from the street:
  - Vortices in the wake of bridge pillars (visualised by water level decrease)
  - Vortices on the corner of houses (visualised by the movement of leaves)
  - Washbasin (visualised by water level)

Vortex detection criteria

- Why the vortices are coherent (does not change much in space and time)?

Chakraborty2005, Haller2005 gives the mathematical proof

Illustration:



- How to find vortices in 3D flowfields?

To select quantities which are related to rotating motion

Vortex detection criteria

Vorticity magnitude

$|\text{rot } \mathbf{v}| > \text{vort}_{th}$ is defined as vortex

- Efficient for free shear flows
- Can not distinguesh between shear generated and rotation related vorticity

Vortex detection criteria (local flow description)

for incompressible (solenoidal) flows

Velocity gradient tensor

$$A_{ij}\partial_j u_i$$

Shear tensor

$$S_{ij}=\frac{1}{2}\left(A_{ij}+A_{ji}\right)$$

Vorticity tensor

$$\Omega_{ij}=\frac{1}{2}\left(A_{ij}-A_{ji}\right)$$

Characteristic equation:

$$0=\lambda^3+\lambda Q+R$$

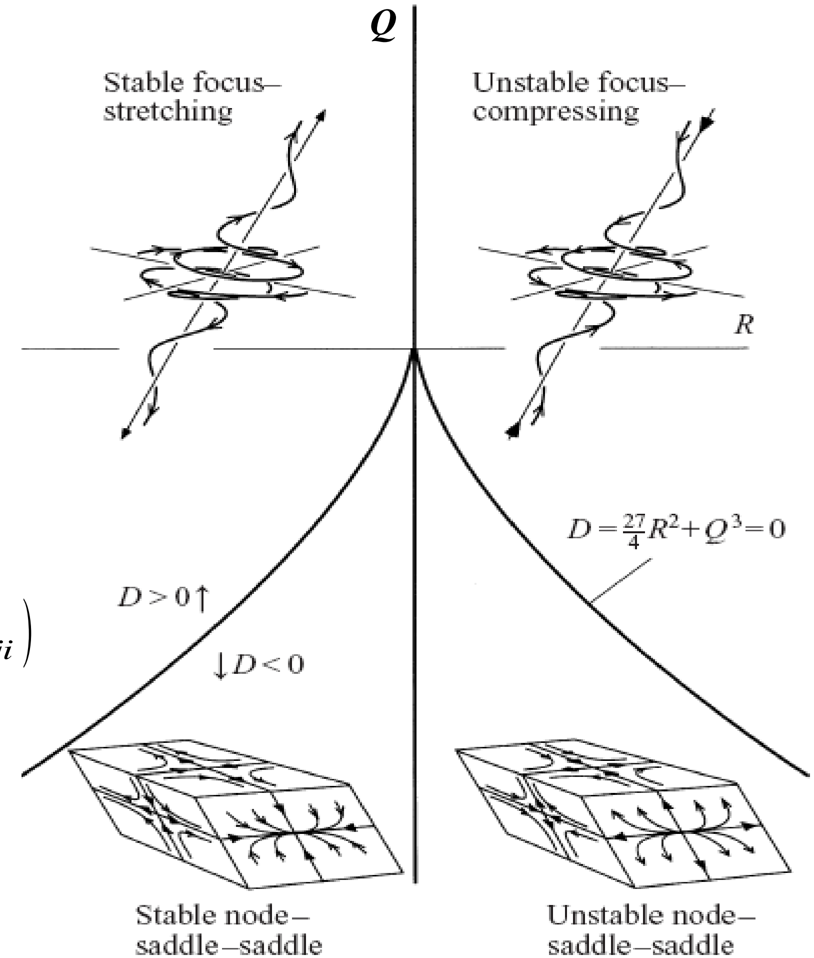
Second scalar invariant (Q):

$$Q=-\frac{1}{2}A_{ij}A_{ji}=\frac{1}{2}\left(\Omega_{ij}\Omega_{ji}-S_{ij}S_{ji}\right)$$

Third scalar invariant (R):

$$R=-\frac{1}{3}A_{ij}A_{jk}A_{ki}$$

Galilean invariant definition



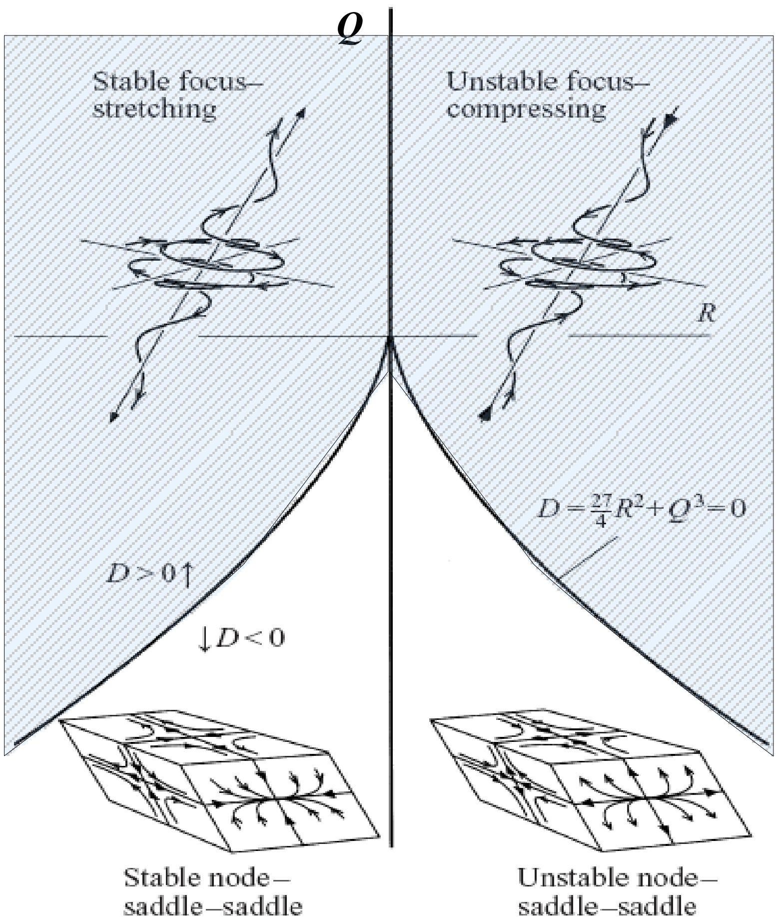
from: (Chacin2000)

Vortex detection criteria

D criteria

- Regions of $D>D_{th}>0$ is defined as the vortex

Everything rotating is identified as a vortex



Vortex detection criteria

Q criteria *(Hunt1988)*

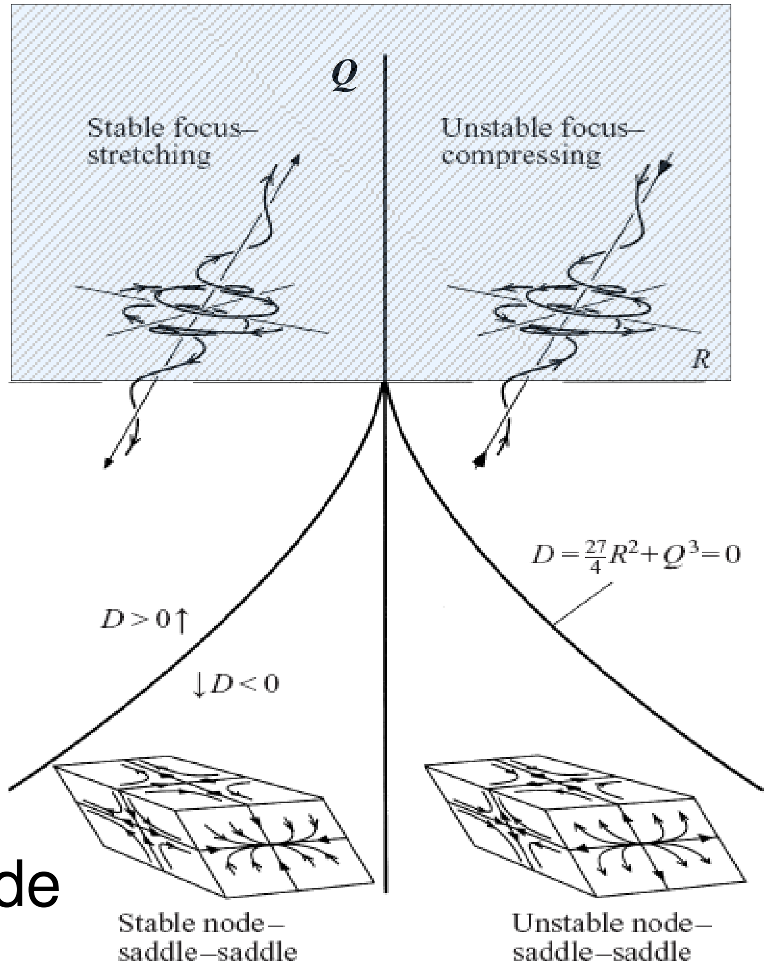
- Regions of $Q>Q_{th}>0$ with local pressure minima is defined as vortex
- Only a fraction of the rotating fluids is defined as a vortex

$$Q=-\frac{1}{2}A_{ij}A_{ji}=\frac{1}{2}\left(\Omega_{ij}\Omega_{ji}-S_{ij}S_{ji}\right)$$

Vorticity dominance

Q>0 means vorticity dominance

- *Charkaborty2005* showed that beside D>0 Q>0 is needed for coherence



Vortex detection criteria

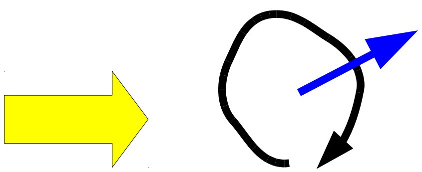
Q criteria *(Hunt1988)*

Q is the source term in the Poisson equation for pressure

Pressure equation:

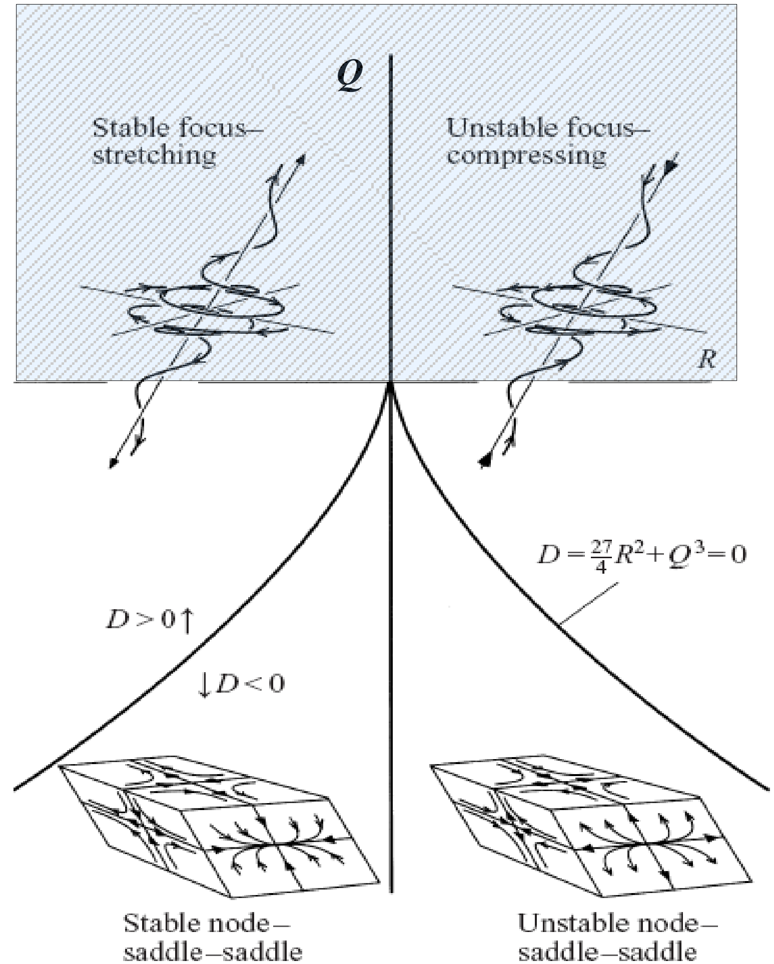
$$\Delta p = 2\rho Q$$

Source term



$$\frac{\partial p}{\partial n} < 0$$

Pressure is lower in the centre of the vortex



Vortex detection criteria

λ_2 *(Jeong1995)*

A criteria to find local pressure minimas because of vortices

Take the gradient of the NS-equation:

$$a_{i,j} = -\frac{1}{\rho}p_{,ij} + \nu u_{i,jkk},$$

Decompose the LHS to symmetric and anti-symmetric:

$$a_{i,j} = \underbrace{\left[\frac{DS_{ij}}{Dt} + \Omega_{ik}\Omega_{kj} + S_{ik}S_{kj}\right]}_{\text{symmetric}} + \underbrace{\left[\frac{D\Omega_{ij}}{Dt} + \Omega_{ik}S_{kj} + S_{ik}\Omega_{kj}\right]}_{\text{antisymmetric}}.$$

The antisymmetric part is the vorticity transport equation.
Let us consider the symmetric part!

$$\frac{DS_{ij}}{Dt} - \nu S_{ij,kk} + \Omega_{ik}\Omega_{kj} + S_{ik}S_{kj} = -\frac{1}{\rho}p_{,ij}.$$

Vortex detection criteria

$$\frac{DS_{ij}}{Dt} - \nu S_{ij,kk} + \Omega_{ik}\Omega_{kj} + S_{ik}S_{kj} = -\frac{1}{\rho}p_{,ij}.$$

In-plane pressure local minimum are related to this Hessian
BUT: Unsteady strain and viscosity created
minimas are not interesting, consider only

$$S^2 + \Omega^2$$

Local in-plane minimum can be determined
from this second eigenvalue

The criteria:

- Regions of $\lambda_2 < \lambda_{2,th} < 0$ is defined as vortex

Local pressure minima in plane due to vortical structure

Vortex detection criteria

Further different criteria and they comparisons:

Jeong1995
Wu2005
Dubief2000
Haller2005
Chakraborty2005
Kollar2007

Example: $Q = -\frac{1}{2}\left(\lambda_1 + \lambda_2 + \lambda_3\right)$

Nicest CS example:
Jeong1997

Application of vortex detection

3 examples:

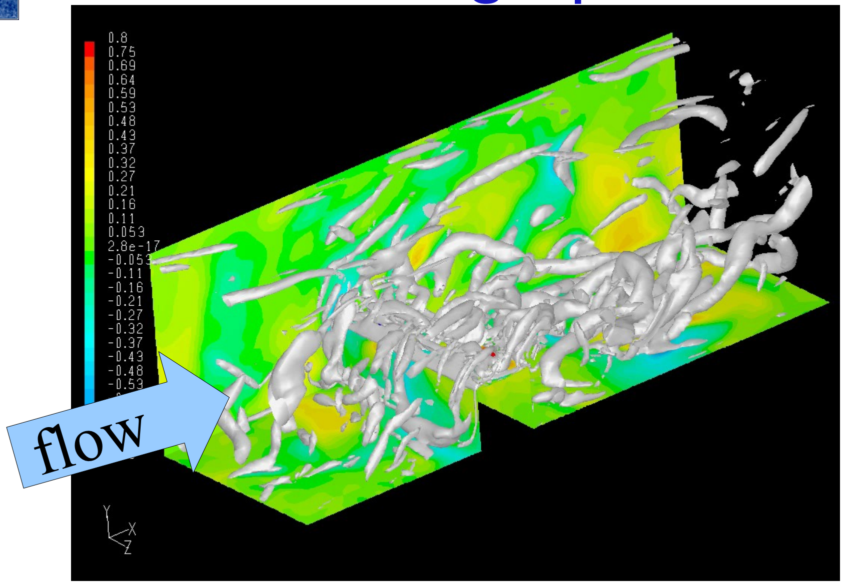
- Visualisation, image processing
- Conditional averaging
- Tracking

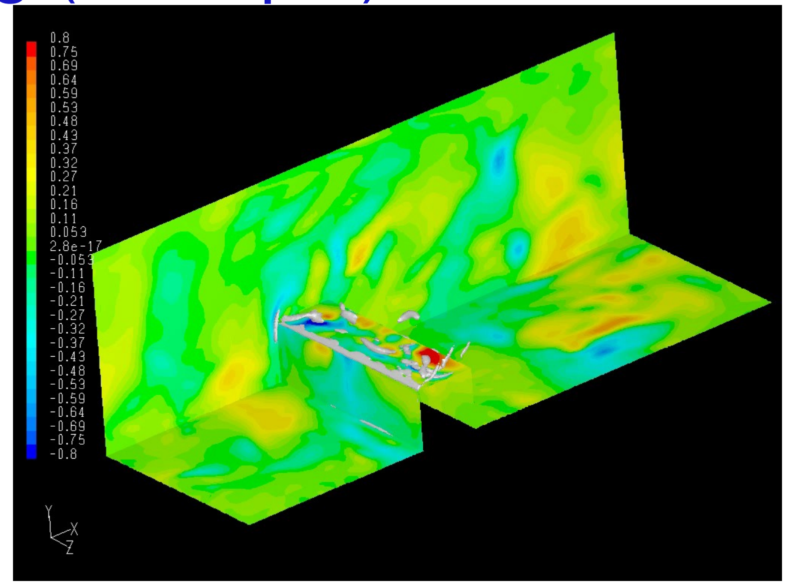
Image processing

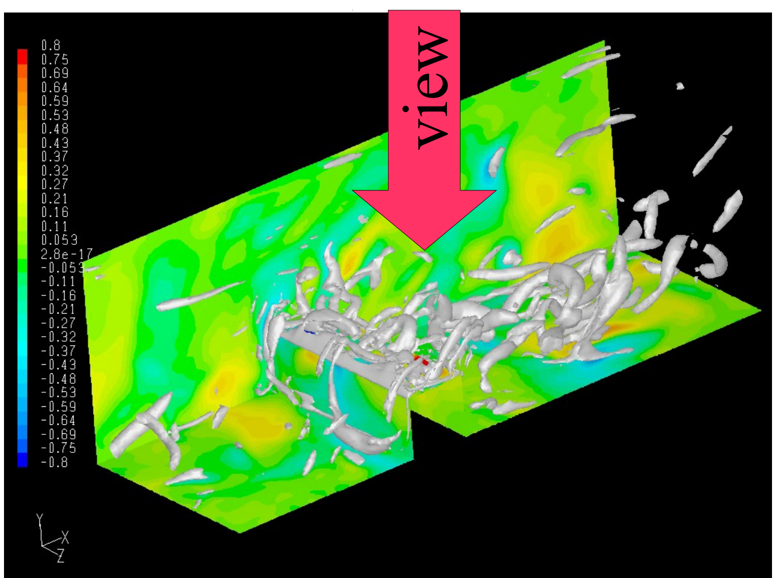
Technology:

1) Create movies of the temporal evolution of the vortices with different thresholds, and different viewpoints
2) Find “well known” features
3) Quantify what you can
4) Compare to possibly existing theory

Image processing (example)







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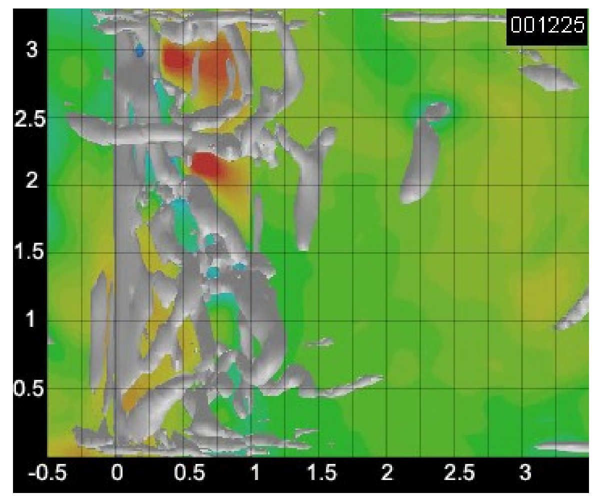
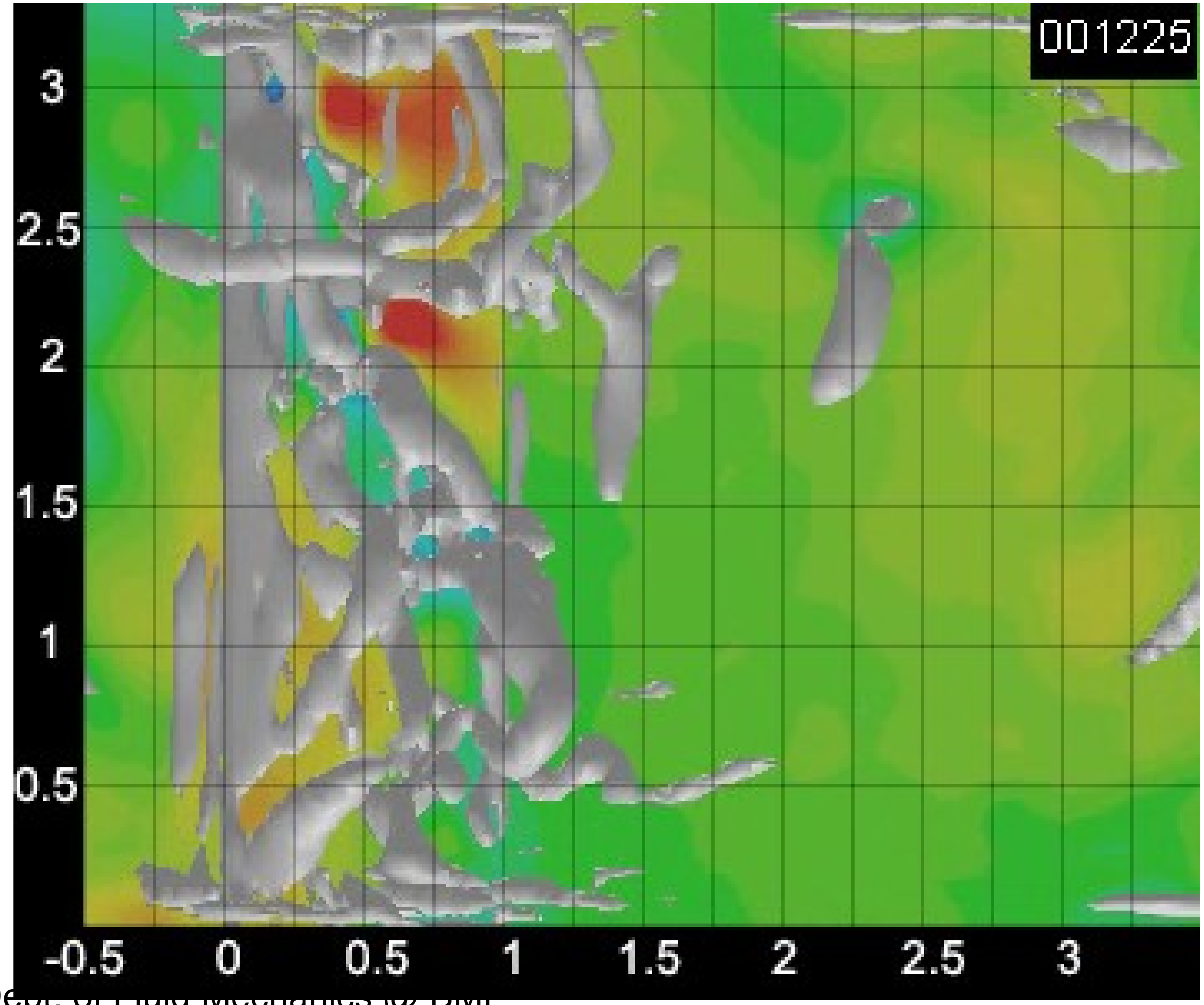


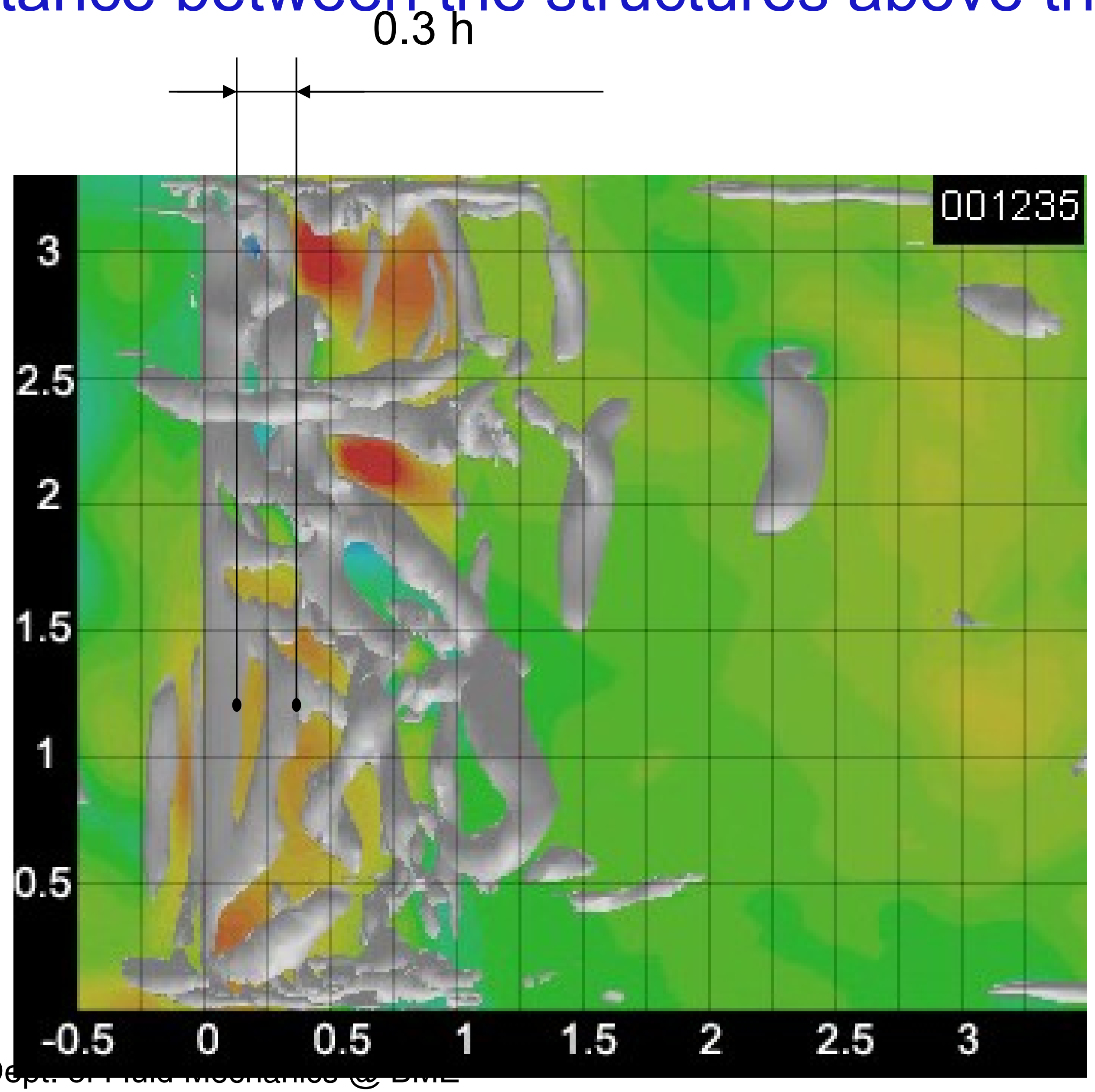
(Lohász2006)

Distance between the structures above the rib

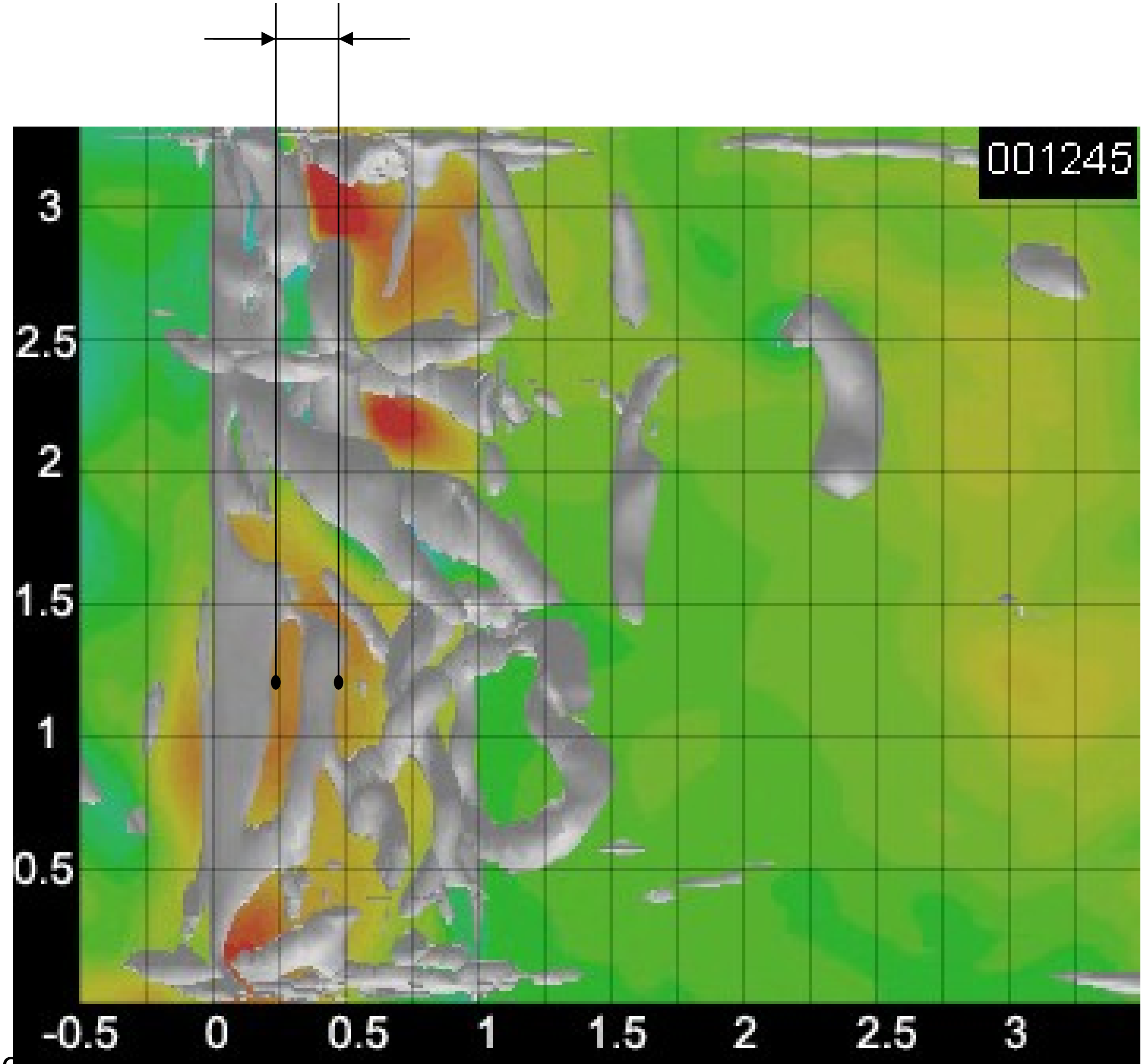
Changes between 0.2-0.3h



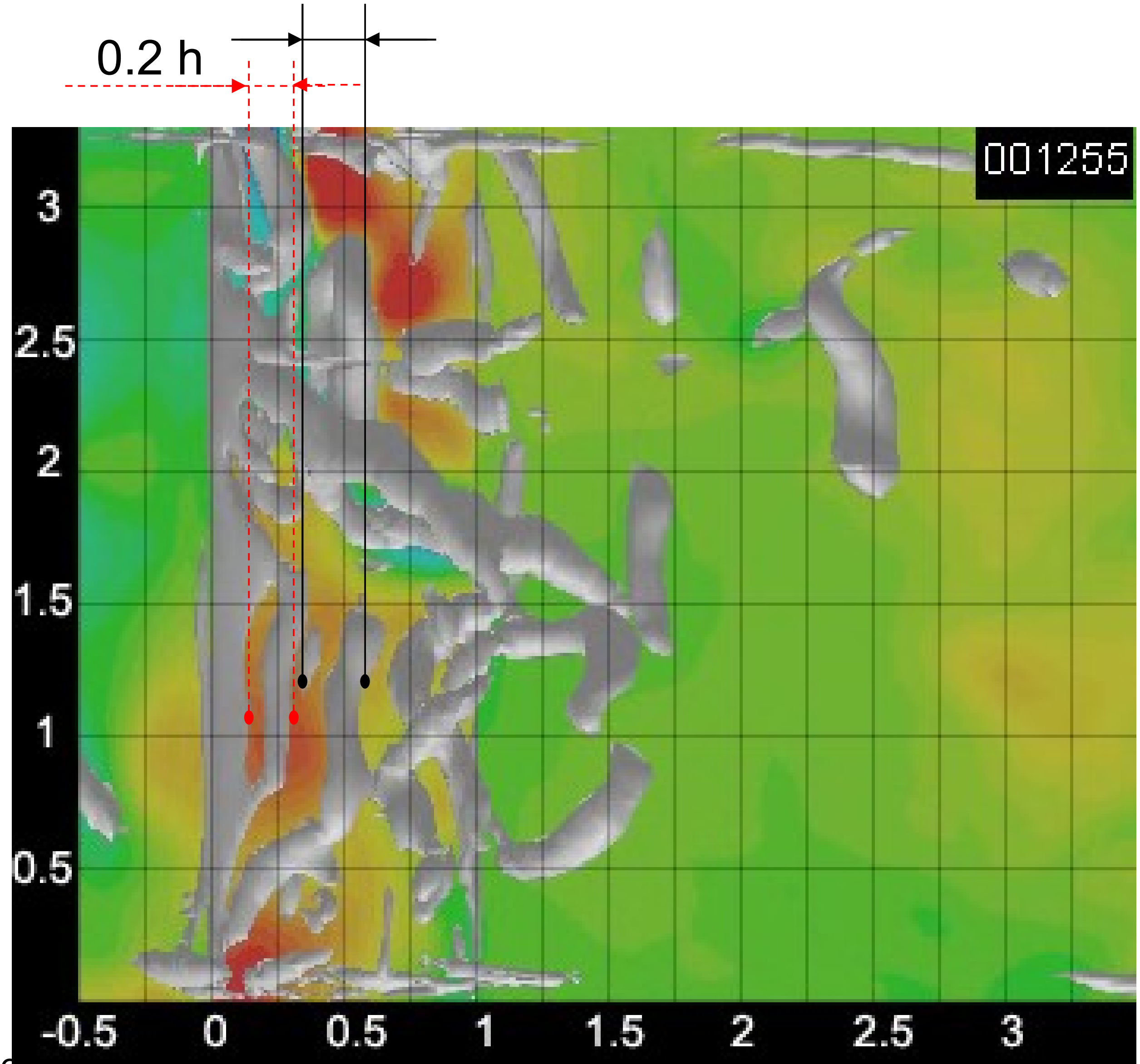
Distance between the structures above the rib



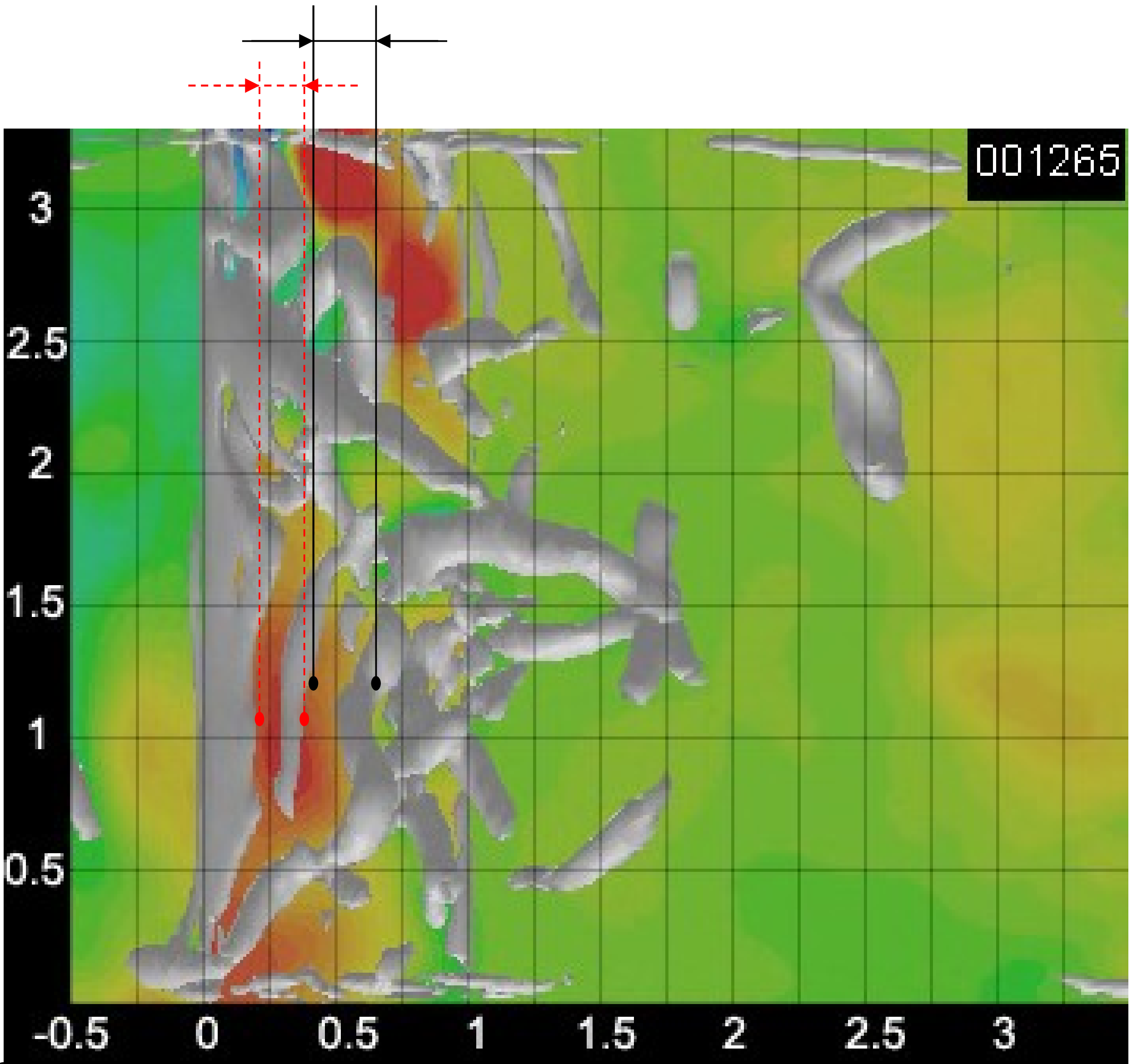
Distance between the structures above the rib



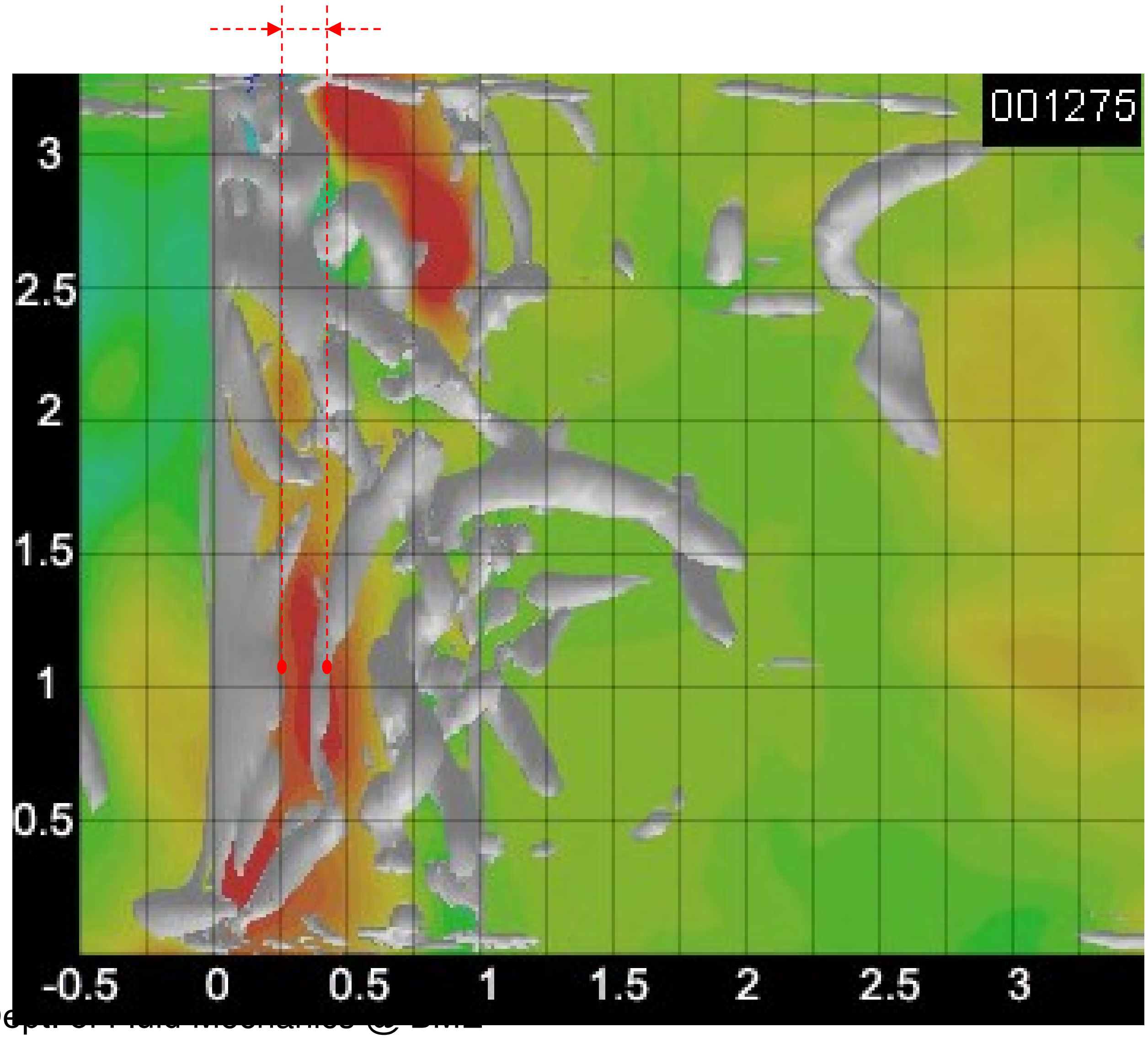
Distance between the structures above the rib



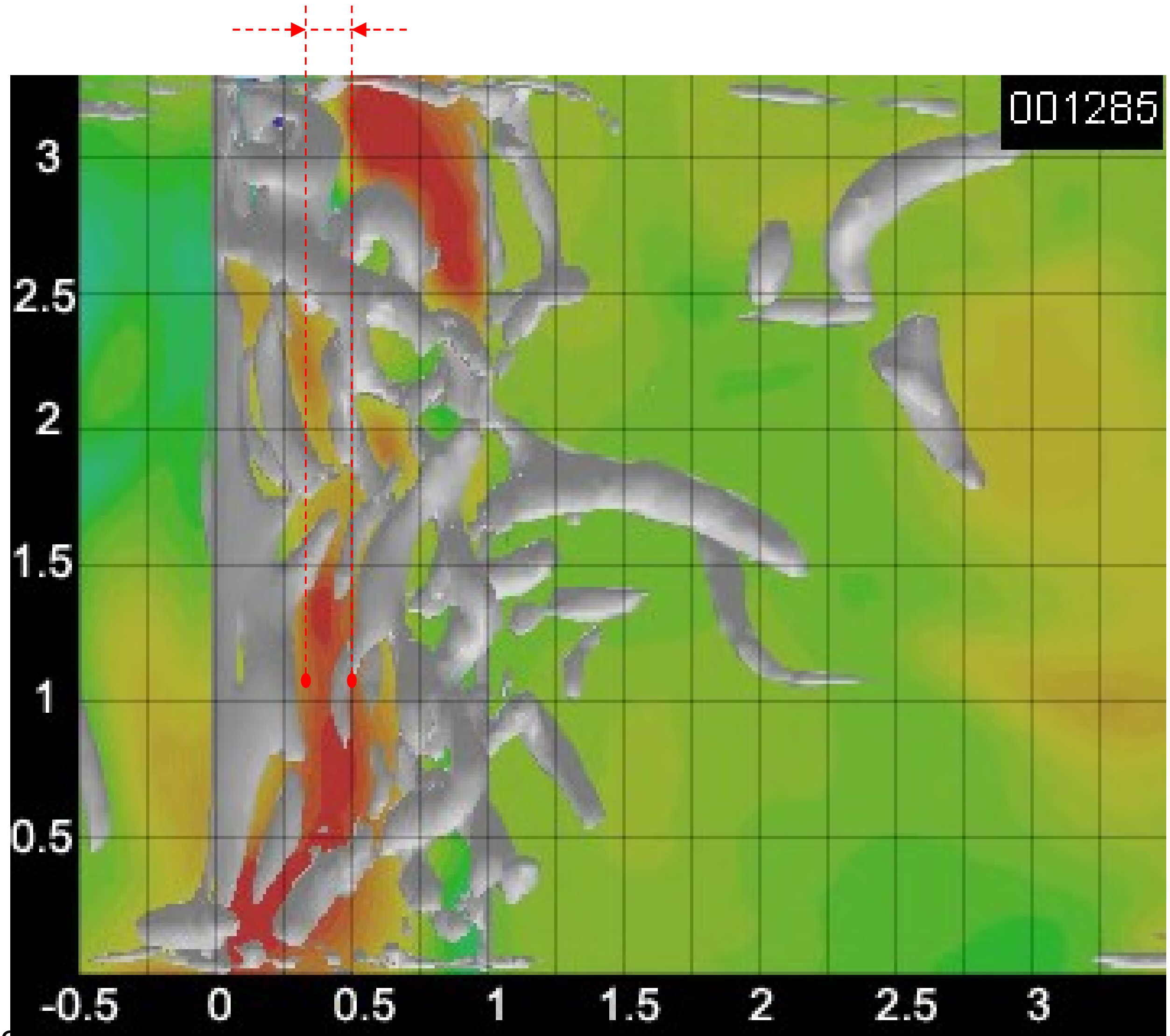
Distance between the structures above the rib



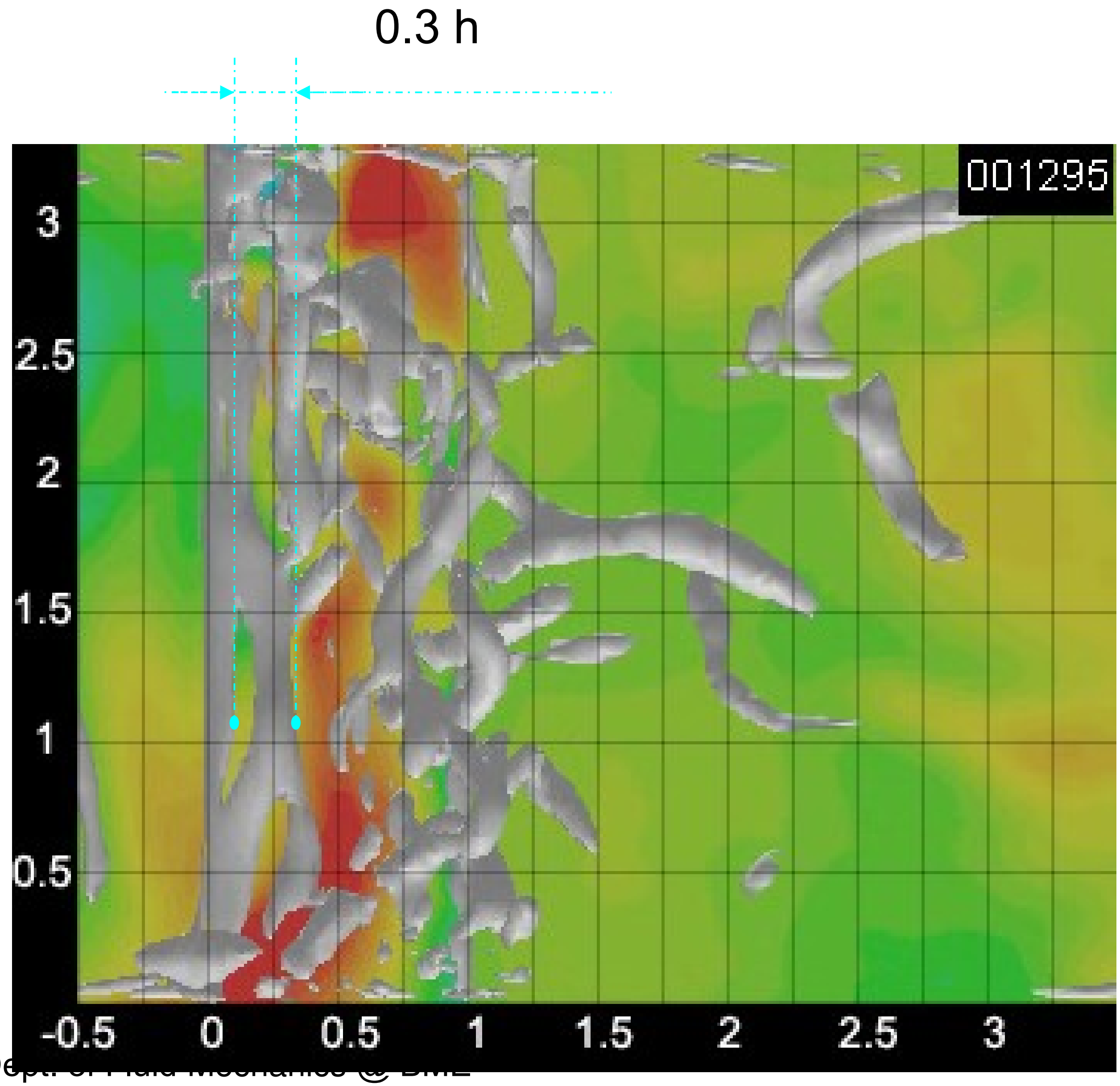
Distance between the structures above the rib



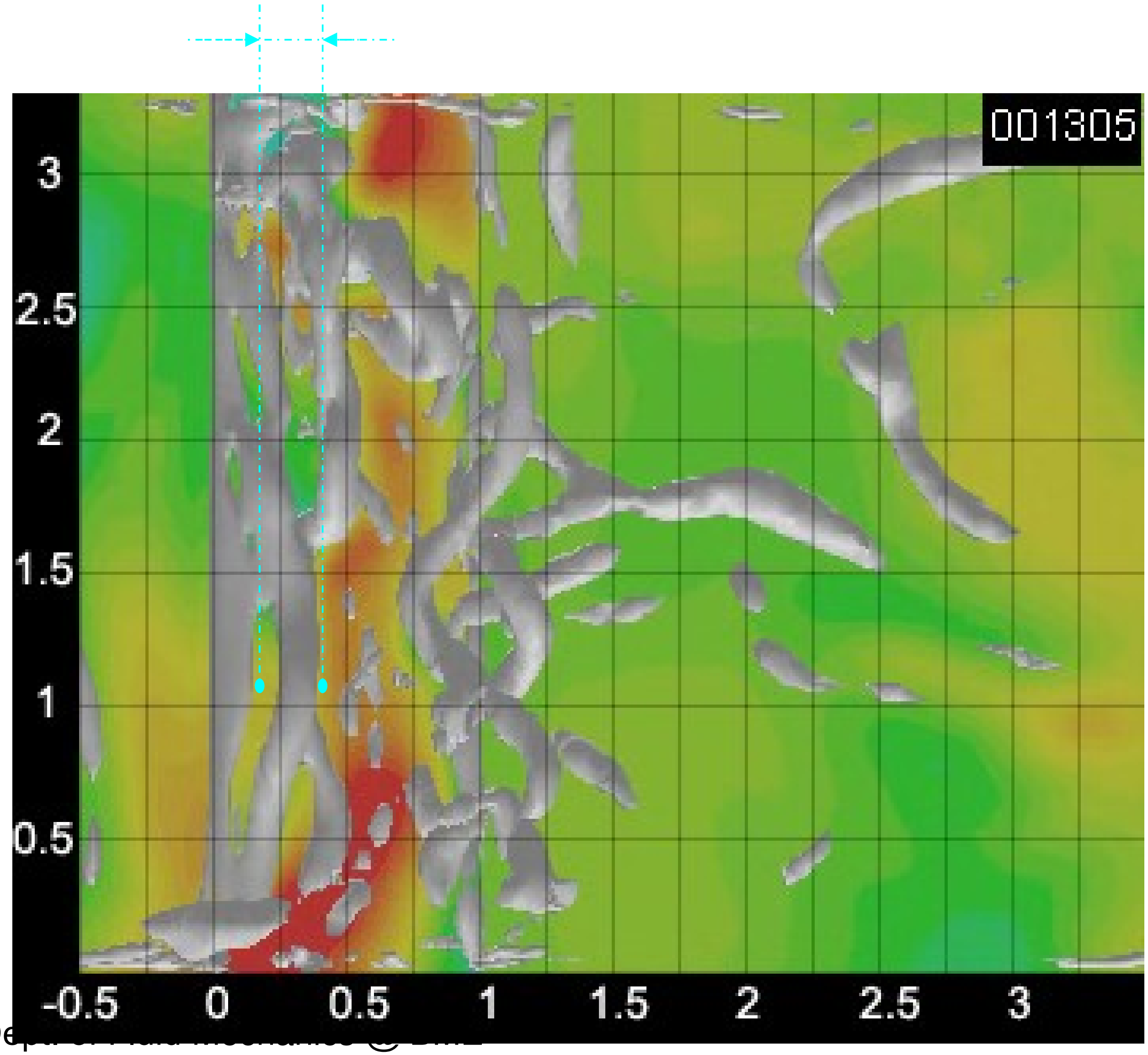
Distance between the structures above the rib



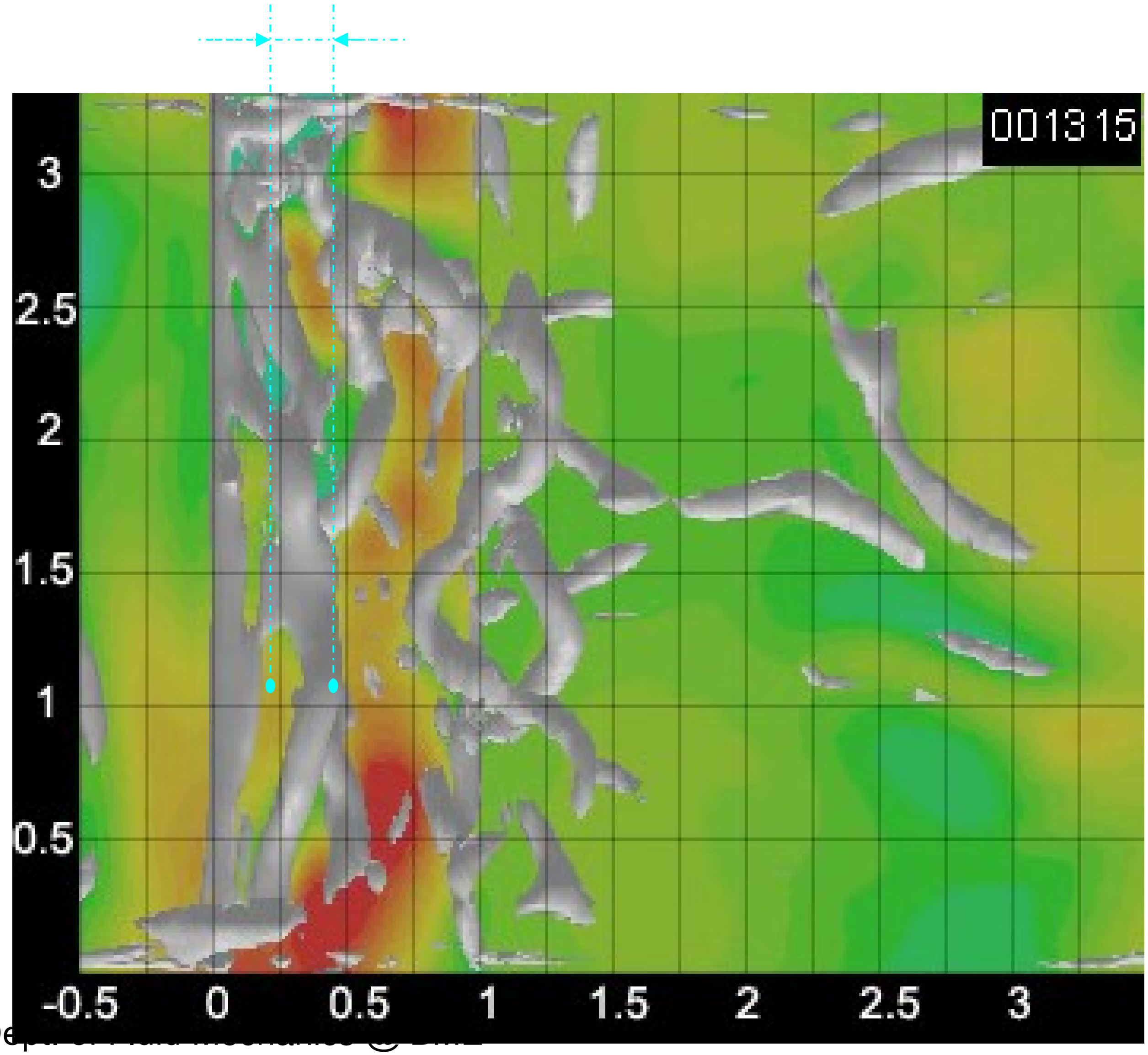
Distance between the structures above the rib



Distance between the structures above the rib



Distance between the structures above the rib



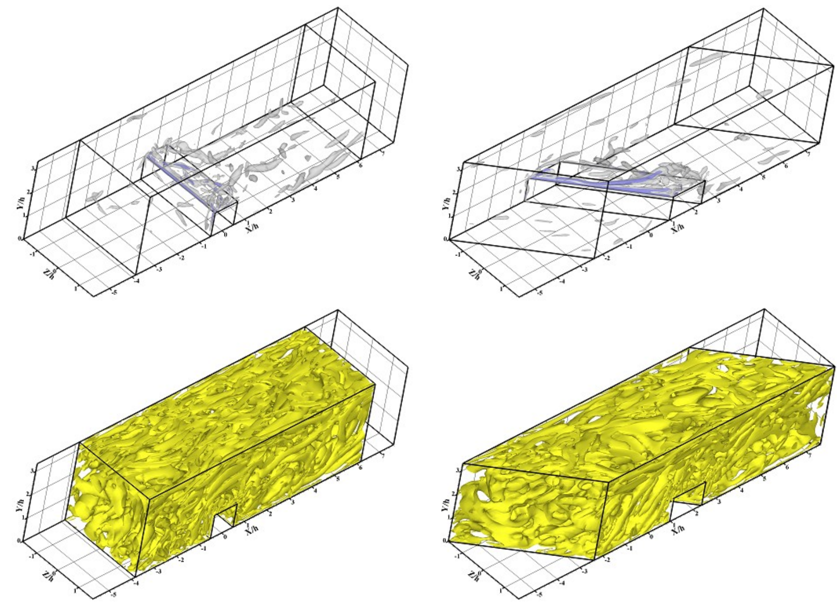
The conditional averaging

Indicator function:

$$I_\alpha \doteq \begin{cases} 1 & Q(\mathbf{x},t) \in Q_\alpha \\ 0 & Q(\mathbf{x},t) \notin Q_\alpha \end{cases}$$

The classes:

$$Q_{I.} \doteq \{x : x \in \mathbb{R} \wedge x < 0\}$$
$$Q_{II.} \doteq \{x : x \in \mathbb{R} \wedge 0 < x < 200\}$$
$$Q_{III.} \doteq \{x : x \in \mathbb{R} \wedge 200 < x < 1500\}$$
$$Q_{IV.} \doteq \{x : x \in \mathbb{R} \wedge 1500 < x \}$$



Conditional averaged variable:

$$\langle \varphi \rangle^\alpha \doteq \frac{\langle \varphi I_\alpha \rangle}{\langle I_\alpha \rangle}$$

Mean value:

$$\langle \varphi \rangle = \sum_\alpha \langle \varphi \rangle^\alpha \langle I_\alpha \rangle$$

Pressure cross correlation:

$$\langle p'^2 \rangle = \langle P^2 \rangle - \langle P \rangle \langle P \rangle$$

(Lohász2005)

Deviation:

$$\Delta^\alpha \varphi = \left(\langle \varphi \rangle^\alpha - \langle \varphi \rangle \right)$$

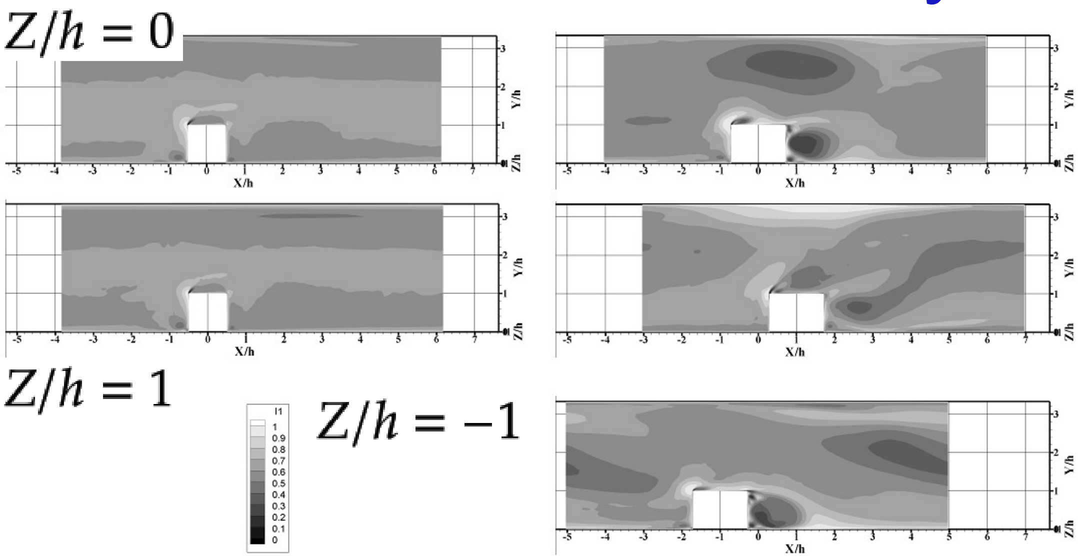
Probability of the classes

Class	Definition	90° Case	45° Case
I.	$Q < 0$	0.6	0.597
II.	$0 < Q < 200U_b^2/D_h^2$	0.376	0.388
III.	$200U_b^2/D_h^2 < Q < 1500U_b^2/D_h^2$	0.019	0.014
IV.	$Q > 1500U_b^2/D_h^2$	0.0008	0.0009
III.+IV.	$Q > 200U_b^2/D_h^2$	0.02	0.015

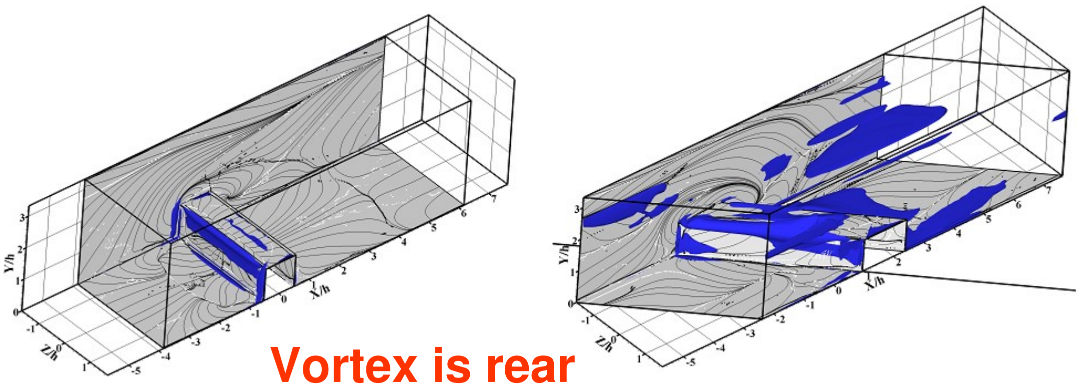
More background than vortex for both cases

Intense vortices are more probable for perpendicular rib

Location of low vorticity dominance regions



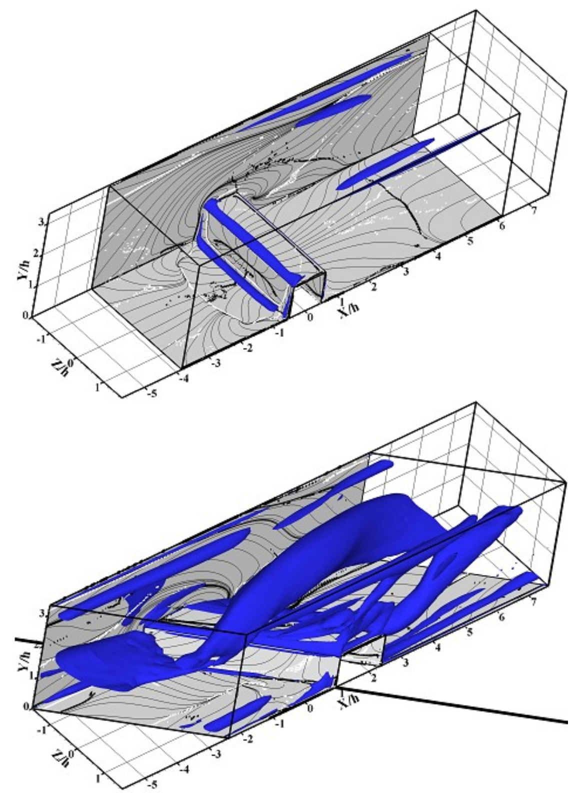
90% probability isosurface of $Q < 0$ class ($\langle I_{I.} \rangle$).



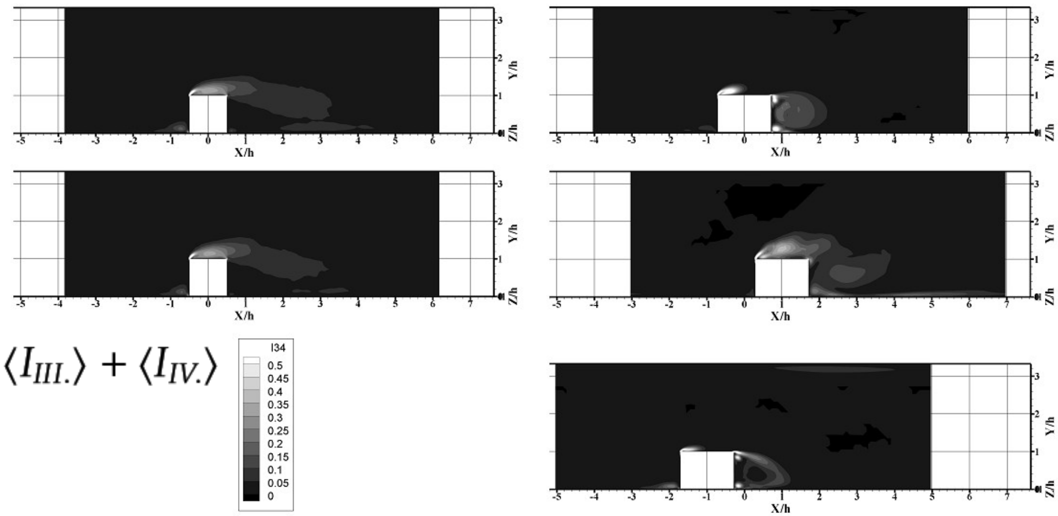
Vortex is rear

Vortex is probable

50% probability isosurface of $Q < 0$ class ($\langle I_{I.} \rangle$).



Location of the intense vortices



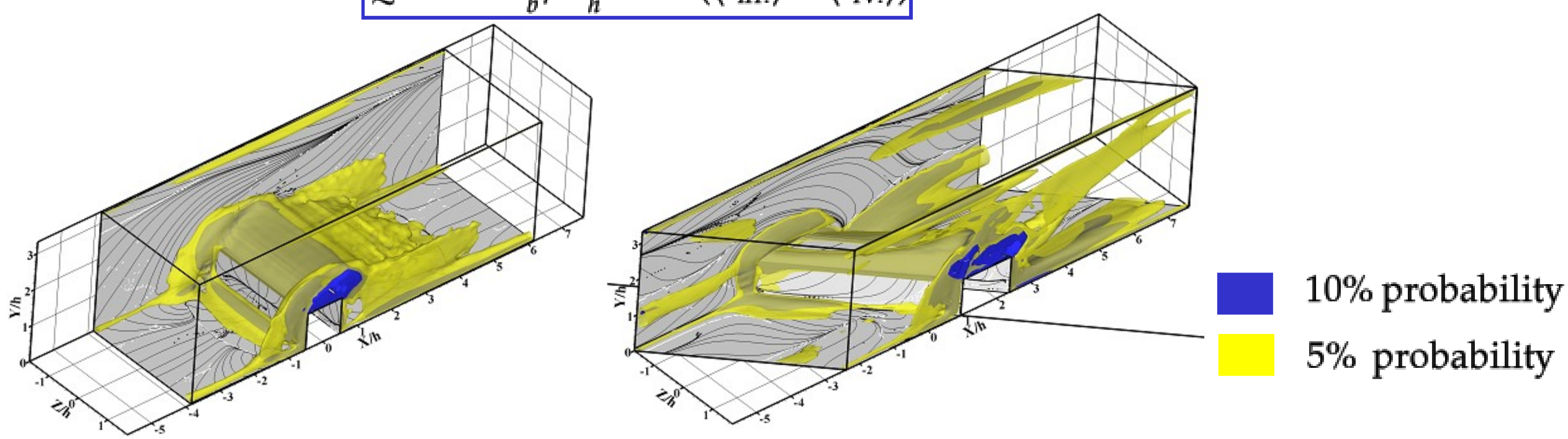
$Z/h = 0$

$Z/h = 1$

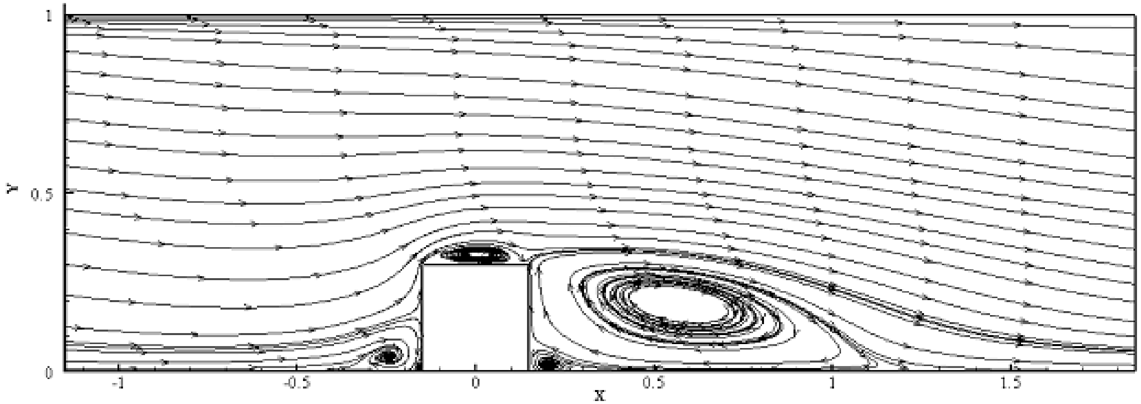
$Z/h = -1$

- Leading edge of the rib
- Shear layer of 90° rib
- Wake of the 45° rib

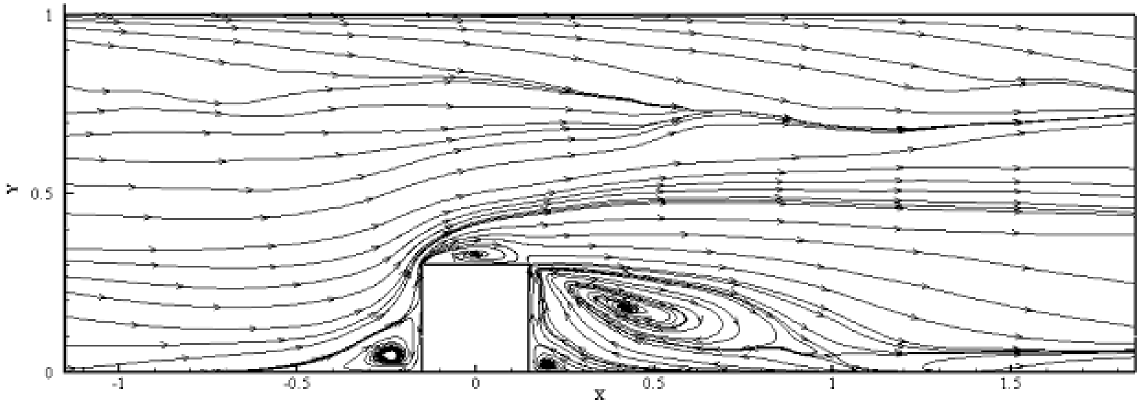
$Q > 200U_b^2/D_h^2$ class $(\langle I_{III.}\rangle + \langle I_{IV.}\rangle)$



Difference in the path of average fluid and the vortices



Average fluid



Intense vortices (34 class)

- The path can be different
- Reattachment of vortices is more upstream

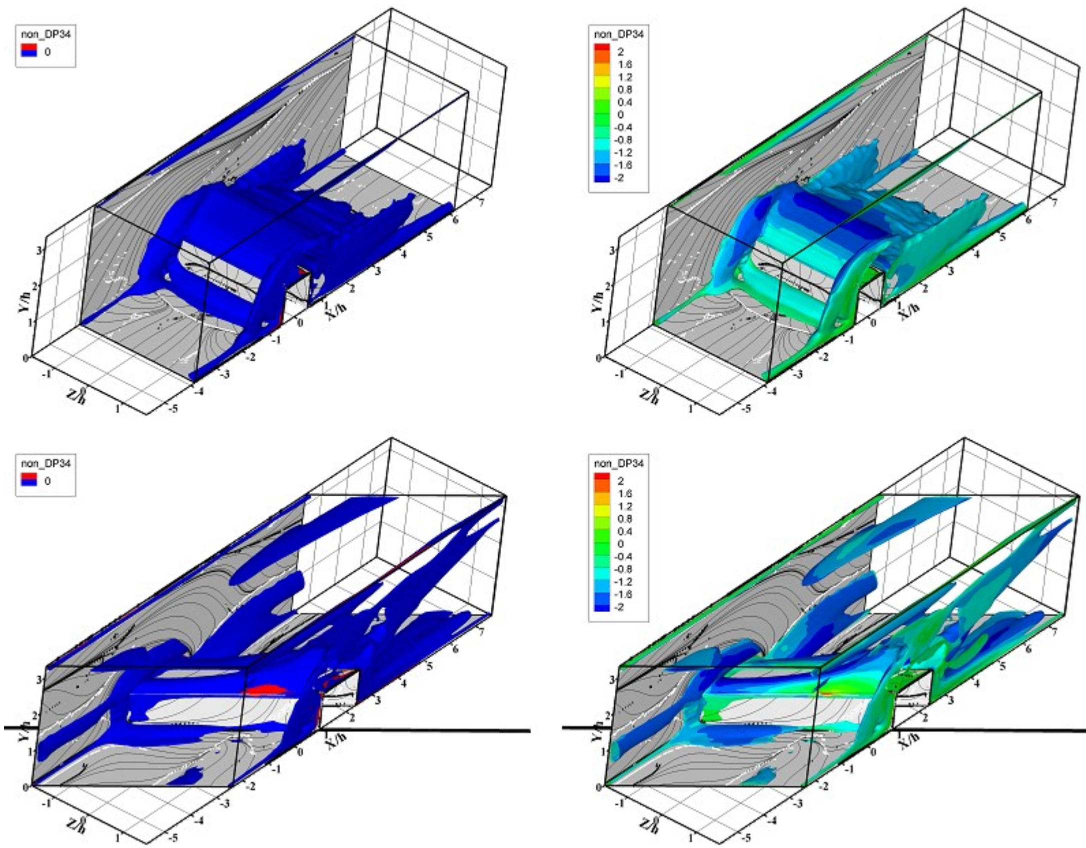
The pressure deviation

Volume average of the pressure deviation for each class for the higher than 5% probability regions

Class	$\Delta^{\alpha}P$ 90°	$\Delta^{\alpha}P$ 45°	90°	45°
I.	0.208	0.302	1	1
II.	−0.254	−0.391	0.999	0.996
III.	−1.083	−1.435	0.084	0.063
IV.	−1.376	−1.201	0.003	0.003
III.+IV.	−1.134	−1.189	0.085	0.064

Size of the higher than 5% probability regions

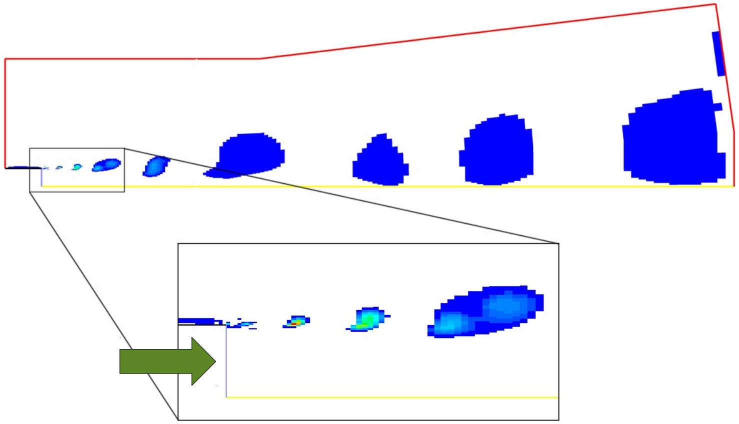
Isosurface of $\langle I_{34} \rangle = 0.05$
coloured by $\langle \Delta^{34} P \rangle / \sqrt{\langle p'^2 \rangle}$



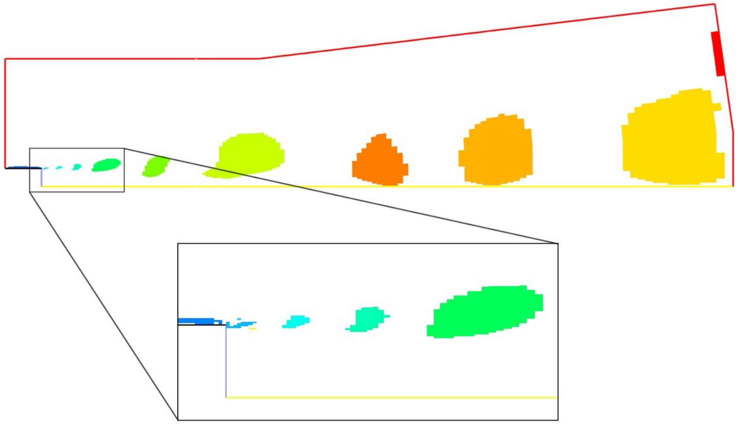
Vortex tracking

- Vortices needs to be identified separatly
  - The educted region needs to be divided into disjunct sets
- Needed for the quantitative investigation of the interactions

The complete educed region



Vortices with indices



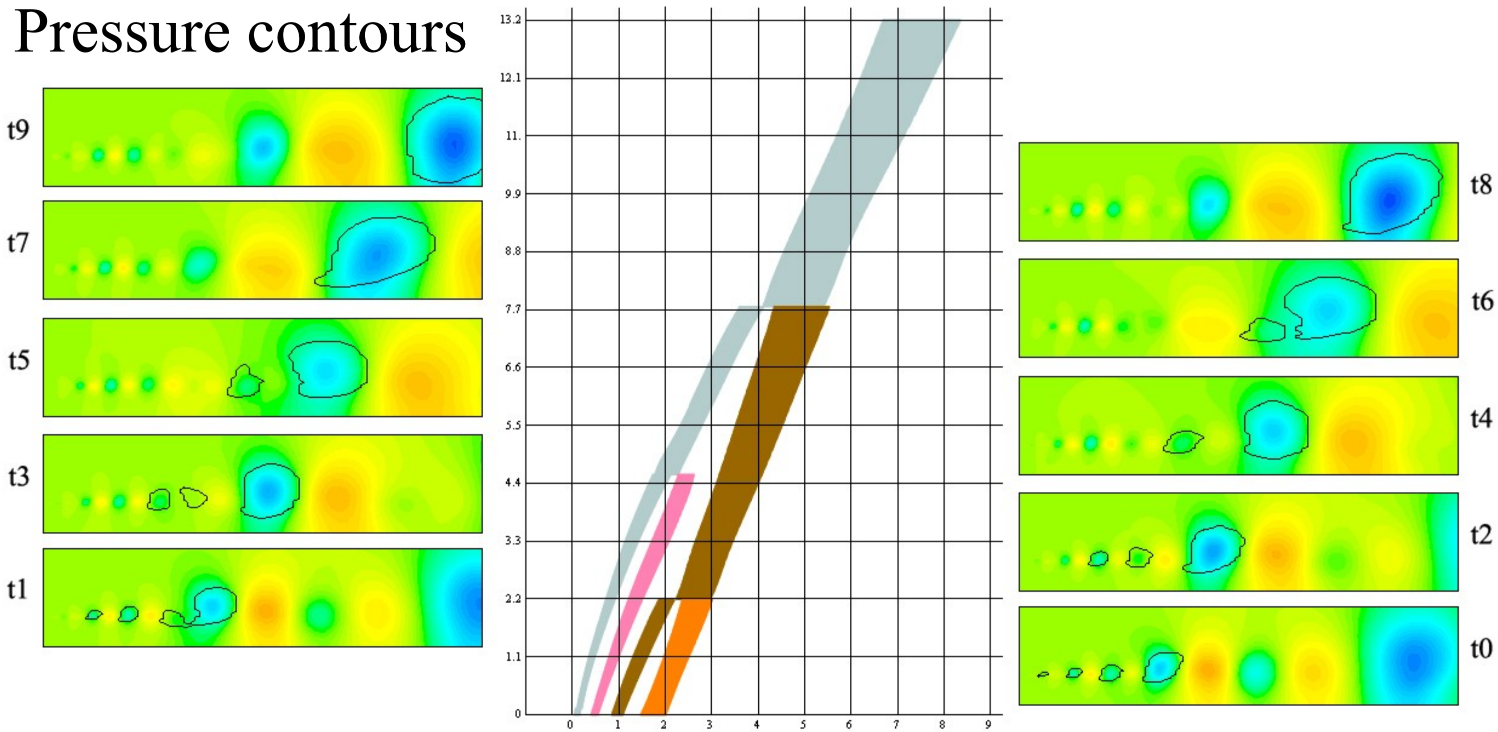
Example of an axisymmetric jet

(Nyers2008)

Vortex tracking

Application example:

Position and size of the vortices

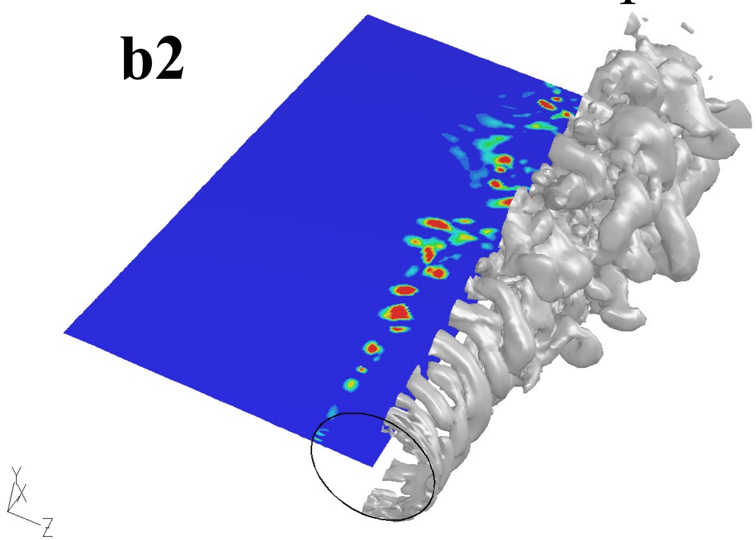


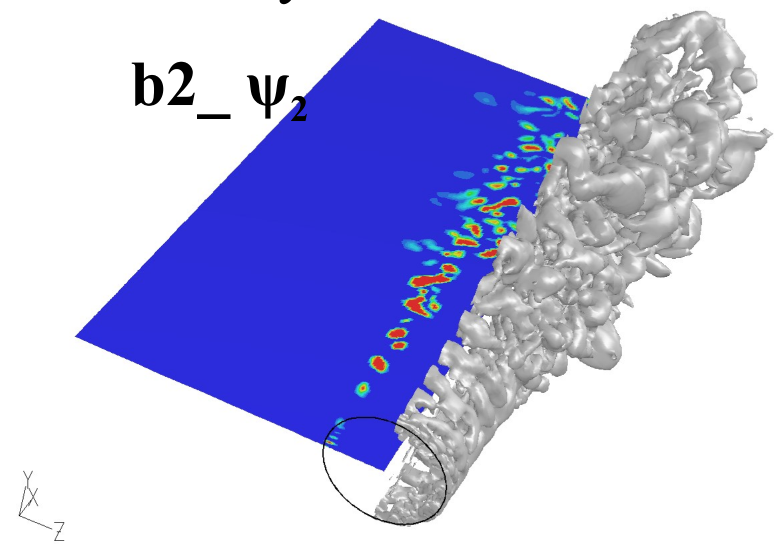
Merging (quadrupole source) process can be investigated in detail!

Worries

- Are the coherent structures of an LES accurate?
  - Can be wrong beside good predicted local fluctuating quantities
  - At least model/grid sensitivity should be checked

Example: Grid sensitivity





$[D^2/U^2_0]$

(Tóth2008)

Statistics:
TKE maximum value is increased, maximum position is more upstream

Structures:
Azimuthally more fragmented vortices

Summary

- If you have an "accurate" LES try to post-process in detail
- Try to find relation between vortices and "simple" statistics
- Perhaps you can find a way to control some phenomena trough vortices
- Good luck!

Thank you for your attention!