7. Atmospheric flows

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Energy balance of a moving volume of air in the atmosphere

The atmosphere feels colder at the top of high mountain. Cold air does not flows down to sea level. Why?

The cold air flow would warm up due to the increasing pressure. (We cannot observe this phenomenon in any laboratory experiments.)

\[ du = \Delta E + \Delta H \]
change of the internal energy of a unit mass of air.

\[ \Delta E : \]
work due to:
- reversible compression;
- viscous dissipation.

\[ \Delta H : \]
heating due to:
- turbulent transport,
- radiation,
- latent heat release (phase change).

The change of the thermodynamic state in a rapid vertical flow of dry air is dominated by the reversible compression work, therefore it is isentropic.

The potential temperature: \( \Theta \)

For isentropic flows:

\[ \frac{T}{T_0} = \left( \frac{P}{P_0} \right)^{k-1} = \left( \frac{\rho}{\rho_0} \right)^{k-1} \]

in which \( k \) is the ratio of specific heats, thus

\[ \frac{k-1}{k} = \frac{c_p - c_v}{c_v} = \frac{R}{c_p} \]

The potential temperature of the atmospheric air in a point characterized by temperature \( T \) and pressure \( p \) is defined as:

\[ \Theta = T \left( \frac{10^5 \text{Pa}}{p} \right) \frac{R}{c_p} \]

\( \Theta \) would be the temperature if the air parcel would taken down to sea level.
Stable and unstable stratifications

On the basis of the potential temperature profile we can analyze the atmospheric stability:

Hydrostatics

\[ \frac{dp}{dz} = -\rho g \]

In order to solve this we need to find relation between \( \rho \) and \( \rho \).

Barotropic relation:

\[ \rho = f(p) \]

- \( \rho = \text{const.} \) homogenous atm.
- \( \rho = \frac{p}{RT_0} \) isothermal atm.
- \( \rho = \rho_0 \left( \frac{p}{p_0} \right)^m \) polytrophic atm.
- \( \rho = \frac{p}{RT} \), \( T = f(z) \) e.g. \( \Theta \)-const.
Problem #7.1

Please, calculate the pressure profile for a given (linear) temperature profile:

\[ T = T_0 - \gamma z, \]

in which \( \gamma = -\frac{\partial T}{\partial z} \) = const. .

Assume, that in \( z = 0 \): \( T = T_0 \) and \( p = p_0 \) !

The dry adiabatic temperature gradient

We compare the \( T(p) \) relation of a linear temperature model with those of an adiabatic model.

From the linear \( T \) proff of \( \gamma \) gradient we obtained:

The adiabatic relation:

\[
\rho = \rho_0 \left( \frac{T_0 - \gamma z}{T_0} \right)^{\frac{\gamma}{\gamma - 1}} = \left( \frac{p}{p_0} \right)^{\frac{\gamma}{\gamma - 1}} = \left( \frac{T}{T_0} \right)^{\frac{\gamma}{\gamma - 1}} = \left( \frac{p}{p_0} \right)^{\frac{\gamma}{\gamma - 1}}
\]

By comparing the exponents:

\[
\frac{R \gamma_{\text{adab}}}{g} = \frac{R}{c_p}
\]

we obtain the adiabatic temperature gradient:

\[
\gamma_{\text{adab}} = \Gamma = \frac{g}{c_p} = \frac{9.8}{1000} = 9.8 \text{ K/km}
\]

Thus the dry adiabatic \( T \) profile is a linear profile of \( \Gamma \) gradient.

The standard atmosphere

Standard (average) tropospheric temperature gradient:

\[ \gamma = 6.5 \text{ K/km} \]

Reference values:

\[ T_0 = 288.15 \text{ K} \]

\[ p_0 = 1.01325 \times 10^5 \text{ Pa} \]
Unstable stratification

During the summer, when the surface is heavily heated:

Thermal convection:

\[ \text{Constant potential temperature.} \]

The pressure gradient

The vertical component:

\[ \frac{\partial p}{\partial z} = -\rho g \approx -12 \frac{\text{Pa}}{\text{m}} \]

The vertical component:

\[ \frac{\partial p}{\partial x} = 26 \text{ hPa} = 0.0026 \frac{\text{Pa}}{\text{m}} \]

This high anisotropy causes problem in numerical solutions.

http://www.weatheronline.co.uk/map/vor/euro/d.htm

Buoyancy and the Boussinesq model

The vertical component of the equation of motion:

\[ \frac{\partial w}{\partial t} = -\frac{\partial p}{\partial z} - \rho g \]

We define the hydrostatic state:

\[ 0 = -\frac{\partial p}{\partial z} - \rho g \]

We decompose the profiles by using the hydrostatic state:

\[ p = \overline{p} + p' \quad \rho = \overline{\rho} + \rho' \quad T = \overline{T} + T' \]

After subtracting the hydrostatic profile we obtain:

\[ \frac{\partial w}{\partial t} = -\frac{\partial p'}{\partial z} - \rho' g \]

If \( p = \text{const} \):

\[ \frac{\rho'}{\overline{\rho}} = -\frac{T'}{\overline{T}} \]

\[ \beta = \overline{T}^{-1} \]

The density perturbation can be expressed in terms of the cubic heat expansion coefficient \( \beta \).
Acoustic filtering

The density need to depend on the pressure, but we need to eliminate the acoustic waves. (Acoustic effects require very small time stepping.)

The continuity equation for compressible fluid:

\[
\frac{\partial \rho}{\partial t} + \frac{\partial \rho u}{\partial x} + \frac{\partial \rho v}{\partial y} + \frac{\partial \rho w}{\partial z} = 0
\]

\[
\rho = \rho(z)
\]

\[\mathbf{v} \cdot \nabla \rho = 0\]

Since the average density is a function of the altitude, this is a more complex continuity equation we normally use in incompressible flow models.

Coriolis force

Inertial forces in a rotating frame:

- Centrifugal force
- Coriolis force

\[
\mathbf{C} = -2\Omega \times \mathbf{v} = -2\Omega_i \mathbf{j} + 2\Omega_j \mathbf{k} + 2\Omega_k \mathbf{i}
\]

\[
C_i = 2\Omega_j \omega_k - 2\Omega_k \omega_j
\]

\[
C_j = 2\Omega_i \omega_k - 2\Omega_k \omega_i
\]

\[
C_k = 2\Omega_i \omega_j - 2\Omega_j \omega_i
\]

The average of \(\omega_i\) is 0.

\[
C_i = v f
\]

\[
C_j = -u f
\]

\[
C_k = 0
\]

in which \(f = 2 \Omega \sin \phi\)

\(\Omega_i = 0\)

\(\Omega_j = \Omega \cos \phi\)

\(\Omega_k = \Omega \sin \phi\)

\(\phi\) the geographic latitude

Geostrophic wind

Steady flow with the assumptions below:

No convective acceleration: streamlines are approximated with parallel lines. Strain stresses are neglected.

Hydrostatic equilibrium in \(z\) direction.

\[
u_x \frac{\partial p}{\partial x} + v \frac{\partial p}{\partial y} = 0
\]

Thus \(\bar{v}_x \cdot \nabla p = 0\)

The pressure gradient is perpendicular to the direction of the equilibrium flow.

Self-consistent specification of the boundary conditions.
Gradient wind

Same as the geostrophic wind excepting that circular streamlines are assumed.

The centrifugal must be taken into account:

\[
\begin{align*}
\frac{\partial u}{\partial x} + v_y &= \frac{1}{\rho} \frac{\partial p}{\partial x} - fu_x \\
\frac{\partial v}{\partial y} + v_x &= \frac{1}{\rho} \frac{\partial p}{\partial y} - fu_y \\
0 &= \frac{1}{\rho} \frac{\partial p}{\partial z} - g
\end{align*}
\]

On the North hemisphere:

cyclone

anticyclone

Problem #7.2

a) Calculate the below non-dimensional pressure gradient for a gradient wind in cylindrical system of coordinates:

\[
\frac{\partial \rho}{\partial r} = \frac{\rho f v_y}{r}
\]

\(r\) is the distance from the center of the cyclone and \(v_y = f(r)\).

b) What is the magnitude of the non-dimensional pressure gradient for a geostrophic wind?

Ekman spiral

Wind direction changes rapidly at the top of the boundary layer (in the outer layer). When approaching the ground the wind direction turns towards the decreasing pressure. The phenomenon is described by a model similar to the geostrophic wind model, but forces rising from the turbulent stresses must be taken into account:

\[
\mathbf{F}_{\text{turb}} = \frac{\partial}{\partial z} \left( \mathbf{v} \frac{\partial \mathbf{v}}{\partial z} \right) + \frac{\partial}{\partial x} \left( \mathbf{v} \frac{\partial \mathbf{v}}{\partial x} \right) + \frac{\partial}{\partial y} \left( \mathbf{v} \frac{\partial \mathbf{v}}{\partial y} \right)
\]

The solution for a medium latitude case on the North hemisphere:
Velocity magnitude in the boundary layer

Two important effects must be taken into account: surface roughness and thermal stratification.

The Monyin-Obuhov profile:

\[
\frac{u}{u^*} = \frac{1}{\kappa} \left( \frac{z}{z_0} + \frac{\beta z}{L} \right)
\]

\[L = \frac{u^2}{\kappa} \left( \frac{z}{z_0} \right) \left( \frac{H_0}{\rho c_L} \right) \]

We can note that, the Reynolds number does not count in atmospheric flows.

The profile of a constant density flow past a flat plate for comparison:

\[
\frac{u}{u_*} = \frac{1}{\kappa} \left( \frac{9 z}{V} \right)
\]

Summary

Most important atmospheric flow related physical phenomena beyond the scope of basic level fluid mechanics:

- Thermal stratification;
- Adiabatic compression and expansion due to vertical flows;
- Variation of density in vertical flows due to the hydrostatic pressure;
- Coriolis force;
- Turbulence in stratified medium;
- Moisture transport and phase changes;
- Surface energy balance involving the radiation heat transport, the heat storage and a number of other complex phenomena.

CFD based atmospheric simulations

Gergely Kristóf Ph.D., Miklós Balogh, Norbert Rácz
29th March 2009.
Advantages of a CFD based model

- Either the surface geometry is described in high details or (alternatively) meso-scale effects are taken into account.
- Bidirectional interface is a source of numerical errors eg. it can cause partial reflection.

Meso scale model

- Higher accuracy
- No limits on geometrical precision
- Flexible meshing
- Advanced turbulent models
- Easy customization
- Advanced pre- and post processing tools

Methodology

ANSYS FLUENT
+ transformation system
+ customized source terms

Mathematical description

\[
\rho = \rho_0 - \rho_b \beta(T - T_0)
\]

\[
\frac{\partial (\rho_0 v)}{\partial t} + \nabla \cdot (\rho_0 v \otimes v) = -\nabla p + \nabla \cdot \tau + \rho_0 \beta \nabla T - \rho_0 \beta \nabla (\rho_b - \rho_0) + F
\]

\[
\frac{\partial \rho_0 f}{\partial t} + \nabla \cdot (\rho_0 f \otimes f) = \nabla \cdot \left(\rho_0 f f \right) - \rho_0 \beta f \nabla (\rho_b - \rho_0) + G
\]

\[
\frac{\partial \rho_0 k}{\partial t} + \nabla \cdot (\rho_0 k \otimes v) = \nabla \cdot \left(\frac{\mu_k}{\sigma_k} \nabla k + \nu \varepsilon \right) + G_k + G_b - \rho_0 \beta \varepsilon S_k
\]

\[
\frac{\partial \rho_0 \varepsilon}{\partial t} + \nabla \cdot (\rho_0 \varepsilon \otimes v) = \nabla \cdot \left(\frac{\mu_\varepsilon}{\sigma_\varepsilon} \nabla \varepsilon + \frac{1}{2} \rho_0 C_1 \frac{\varepsilon^2}{k + \sqrt{\varepsilon}} \right) + C_{\varepsilon} \left(\frac{\mu_b}{\sigma_b} \nabla \varepsilon + \nu \sqrt{\varepsilon} \right)
\]

Customized volume sources
Equilibrium profiles
up to the height of 11 km

\[ T = T_0 - \gamma z \]

\[ p = p_0 \left( \frac{T_0 - \gamma z}{T_0} \right)^\frac{2}{\gamma - 1} \]

\[ T_0 = 288.15 \text{K} \]

\[ \gamma = 1.225 \text{kg/m}^3 \]

\[ \zeta = 10^{-5} \text{m}^3 \]

Standard ISA profile

Approximate profile

Error bound is within 0.4% below 4000 m.

Transformation expressions

\[ T = T_0 + \frac{p}{p_0} \left( \frac{\gamma + 1}{\gamma} \right) \left( \frac{T_0}{p_0} \right)^{\frac{\gamma}{\gamma - 1}} \]

\[ \rho = \rho_0 - \rho + \frac{p}{p_0} \left( \frac{\gamma + 1}{\gamma} \right) \left( \frac{T_0}{p_0} \right)^{\frac{\gamma}{\gamma - 1}} \]

\[ z = -\frac{1}{\gamma} \ln(1 - \frac{z}{z_0}) \]

\[ w = \frac{\rho_0}{\rho} \frac{\gamma - 1}{\gamma} e \]

Summary of source terms

In momentum equation:

\[ S_x = \rho_0 u'v' + \rho_0 \frac{f}{\Omega} \]

\[ S_z = -\rho_0 \frac{f}{\Omega} \]

In the energy equation:

\[ S_y = J (S_{yy} - \rho_0 c_p u'^2) \]

\[ S_y = -\beta \frac{\rho_0}{\rho_0} (f^2 - y) \]

In the transport equation of turbulent kinetic energy:

\[ S_{\kappa} = -C_k \left( \frac{\kappa}{\varepsilon} \right) \frac{\beta g}{\rho_0} u' v' (f^2 - y) \]

\[ S_{\epsilon} = 2 \Omega \cos \theta \]

\[ S_{\epsilon} = 2 \Omega \sin \theta \]

\[ S_{\epsilon} = (1 - \zeta)^2 \]
Related publications


Two validation examples

Comparison with the results of water tank experiments

Experimental setup


Uniformly stratified salt water:

- p: 1.03 - 1.00 g/cm³
- Typical towing speeds:
  - U: 1.15 cm/s

Brunt-Väisälä frequency range:

- N: 1.09 - 1.05 1/s
- Reₘₐₓ: 10⁶ - 10⁷
- Studied obstacle heights:
  - h: 20mm, 40mm
$z(x) = a \exp(-b|x|^\gamma)$

Different parameter set for positive and negative $x$.

Numerical mesh

Gravity waves


Simulation vs. experimental data

Wave length  Amplitude
Comparison of velocity and temperature profiles

Some more application examples
Full scale simulations

Meso scale atmospheric dispersion
Wind speed: 3 m/s
Injection velocity: 5 m/s
Chimney height: 180 m
Standard (stable) temperature profile
Temperature difference: 20 °C

Kelvin-Helmholtz instability

Com domain: 25 km x 5.5 km

Von Kármán vortices behind a volcanic island

First CFD results
Targeted application areas

- Local circulation modeling:
  - Urban heat island convection, ventilation of cities
  - See breeze, Valley breeze
- Power generation and pollution control:
  - Assessment of wind power potential, optimization of wind farms
  - Plumes emitted by cooling towers and chimneys
  - Dispersion of pollutant in the urban atmosphere
- Research of meteorological phenomena:
  - Gravity waves
  - Cloud formation
  - Flow around high mountain
- Simulation of disasters:
  - Large scale fires (e.g. in forest, kind of beer flames)
  - Volcanic plumes