# 7. Atmospheric flows

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## Energy balance of a moving volume of air in the atmosphere

Te atmosphere feels colder at the top of high mountain.
Cold air does not flows down to sea level. Why?

The cold air flow would warm up due to the increasing pressure.
(We cannot observe this phenomenon in any laboratory experiments.)

$du = \delta q + \delta w$ change of the internal energy of a unit mass of air.

$\delta w :$ work due to:
- reversible compression;
- viscous dissipation.

$\delta q :$ heating due to:
- turbulent transport,
- radiation,
- latent heat release (phase change).

The change of the thermodynamic state in a **rapid vertical flow of dry air** is dominated by the **reversible compression work**, therefore it is **isentropic**.

## The potential temperature: $\Theta$

For isentropic flows: $$\frac{T}{T_0} = \left(\frac{p}{p_0}\right)^{\frac{\kappa-1}{\kappa}} = \left(\frac{\rho}{\rho_0}\right)^{\kappa-1}$$

in which $\kappa$ is the ratio of specific heats, thus

$$\frac{\kappa-1}{\kappa} = \frac{c_p - c_v}{c_p} = \frac{R}{c_p}$$

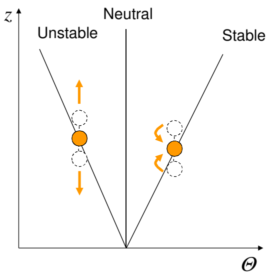
The **potential temperature** of the atmospheric air in a point characterized by temperature $T$ and pressure $p$ is defined as:

$$\Theta = T\left(\frac{10^5\,\mathrm{Pa}}{p}\right)^{R/c_p}$$

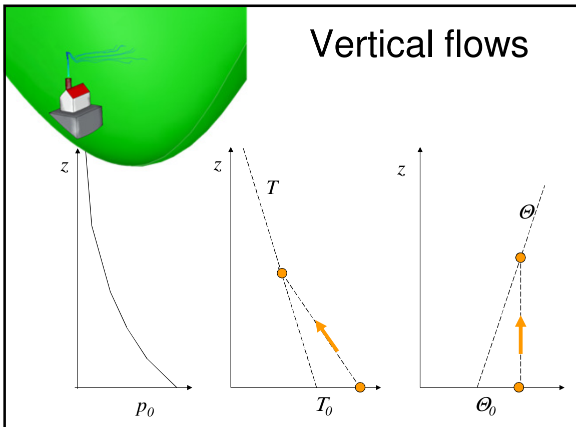
$\Theta$ would be the temperature if the air parcel would taken down to sea level.

## Stable and unstable stratifications

On the basis of the potential temperature profile we can analyze the atmospheric stability:




## Vertical flows




## Hydrostatics

$$\frac{\partial p}{\partial z} = -\rho\, g$$

In order to solve this we need to find relation between $p$ and $\rho$.

Barotrophic relation:

$\rho = f(p)$

- $\rho = \text{const.}$ — homogenous atm.
- $\rho = \dfrac{p}{RT_0}, \quad T_0 = \text{const.}$ — isothermal atm.
- $\rho = \rho_0 \left(\dfrac{p}{p_0}\right)^m, \quad m = \text{const.}$ — polytrophic atm. e.g. $\Theta$=const.
- $\rho = \dfrac{p}{R\overline{T}}, \quad \overline{T} = f(z)$

## Problem #7.1

Please, calculate the pressure profile for a given (linear) temperature profile:

$$\overline{T} = T_0 - \gamma z \quad ,$$

in which $\gamma = -\dfrac{\partial T}{\partial z} = \text{const.}$ .

Assume, that in z=0: $T=T_0$ and $p=p_0$ !

To the solution

## The dry adiabatic temperature gradient

We compare the $T(p)$ relation of a linear temperature model with those of an adiabatic model.

From the linear $T$ profil of $\gamma$ gradient we obtained:

$$p = p_0 \left( \frac{T_0 - \gamma z}{T_0} \right)^{\frac{g}{R\gamma}} \longrightarrow \frac{T}{T_0} = \left( \frac{p}{p_0} \right)^{\frac{R\gamma}{g}}$$

The adiabatic relation:

$$\frac{T}{T_0} = \left( \frac{p}{p_0} \right)^{\frac{R}{c_p}}$$

Aha! T(z) must be also linear for an adiabatic profile!

By comparing the exponents:

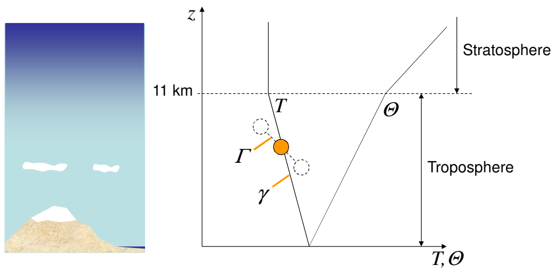
$$\frac{R\,\gamma_{adiab.}}{g} = \frac{R}{c_p}$$

we obtain the adiabatic temperature gradient:

$$\gamma_{adiab.} = \Gamma = \frac{g}{c_p} = \frac{9.8}{1000} = 9.8 \frac{\text{K}}{\text{km}}$$

Thus the dry adiabatic $T$ profile is a linear profile of $\Gamma$ gradient.

## The standard atmosphere



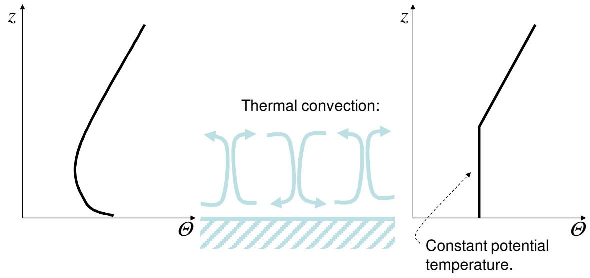

Standard (average) tropospheric temperature gradient: $\gamma = 6.5 \dfrac{\text{K}}{\text{km}} < \Gamma$

Reference values: $T_0 = 288.15\,\text{K}$

$p_0 = 1.01325 \cdot 10^5\,\text{Pa}$

## Unstable stratification

During the summer, when the surface is heavily heated:



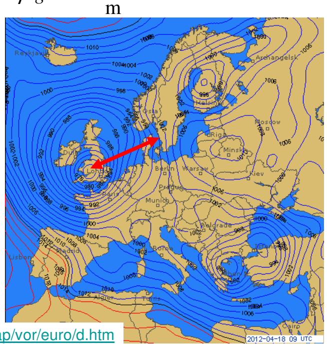

## The pressure gradient

The vertical component: $\frac{\partial p}{\partial z} = -\rho\, g \cong -12 \frac{\text{Pa}}{\text{m}}$

The vertical component:

$$\frac{\partial p}{\partial x} = \frac{26\ \text{hPa}}{1000\ \text{km}} = 0.0026 \frac{\text{Pa}}{\text{m}}$$

This high anisotropy causes problem in numerical solutions.



http://www.weatheronline.co.uk/map/vor/euro/d.htm

## Buoyancy and the Boussinesq model

The vertical component of the equation of motion: $\rho \frac{dw}{dt} = -\frac{\partial p}{\partial z} - \rho\, g$

We define the hydrostatic state: $0 = -\frac{\partial \bar{p}}{\partial z} - \bar{\rho}\, g$

We decompose the profiles by using the hydrostatic state: $p = \bar{p} + p' \quad \rho = \bar{\rho} + \rho' \quad T = \bar{T} + T'$

After subtracting the hydrostatic profile we obtain: $\rho \frac{dw}{dt} = -\frac{\partial p'}{\partial z} - \rho' g$

The density perturbation can be expressed in terms of the cubic heat expansion coefficient $\beta$: $\rho' = -\bar{\rho}\, \beta T'$

If $p = \text{const}$:

$$\frac{\rho'}{\bar{\rho}} = -\frac{T'}{\bar{T}}$$

$$\beta = \bar{T}^{-1}$$

# Acoustic filtering

The density need to depend on the pressure, but we need to eliminate the acoustic waves. (Acoustic effects require very small time stepping.)

The continuity equation for compressible fluid:

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho u}{\partial x} + \frac{\partial \rho v}{\partial y} + \frac{\partial \rho w}{\partial z} = 0$$

$$\rho \cong \bar{\rho}(z)$$

$$\nabla \cdot (\bar{\rho}\, \underline{v}) = 0$$

Since the average density is a function of the altitude, this is a more complex continuity equation we normally use in incompressible flow models.

# Coriolis force

Inertial forces in a rotating frame:
- Centrifugal force — Taken into account in $g$.
- Coriolis force — Effects only moving bodies.

$$\underline{C} = -2\underline{\Omega} \times \underline{v} = -2 \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \Omega_x & \Omega_y & \Omega_z \\ u & v & w \end{vmatrix}$$

$$C_x = 2v\,\Omega_z - 2w\,\Omega_y$$

$$C_y = 2w\,\Omega_x - 2u\,\Omega_z$$

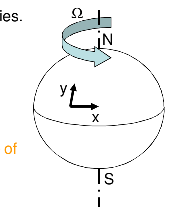
$$C_z = 2u\,\Omega_y - 2v\,\Omega_x$$

the average of w is 0.

the buoyancy force prevails in vertical flows

$$C_x \cong v\,f \qquad C_y = -u\,f \qquad C_z \cong 0$$

in which $f = 2\Omega\, sin\phi$




$$\Omega_x = 0$$

$$\Omega_y = \Omega\, cos(\phi)$$

$$\Omega_z = \Omega\, sin(\phi)$$

$\phi$ the geographic latitude

# Geostrophic wind

**Steady flow** with the assumptions below:

$$0 = -\frac{1}{\rho}\frac{\partial p}{\partial x} + f v_g$$

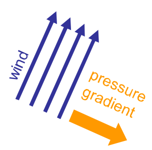
$$0 = -\frac{1}{\rho}\frac{\partial p}{\partial y} - f u_g$$

No convective acceleration: streamlines are approximated with parallel lines. Strain stresses are neglected.

$$0 = -\frac{1}{\rho}\frac{\partial p}{\partial z} - g$$

Hydrostatic equilibrium in z direction.

On the North hemisphere:




$$u_g \frac{\partial p}{\partial x} + v_g \frac{\partial p}{\partial y} = 0 \quad \text{thus} \quad \vec{v}_g \cdot \nabla p = 0$$

The pressure gradient is perpendicular to the direction of the equilibrium flow.

**... self-consistent specification of the boundary conditions.**

# Gradient wind

Same as the geostrophic wind excepting, that **circular streamlines** are assumed.

The centrifugal must be taken into account:

$$u_g \frac{\partial u_g}{\partial x} + v_g \frac{\partial u_g}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + f v_g$$

$$u_g \frac{\partial v_g}{\partial x} + v_g \frac{\partial v_g}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial y} - f u_g$$

$$0 = -\frac{1}{\rho}\frac{\partial p}{\partial z} - g$$

On the North hemisphere:


# Problem #7.2

a) Calculate the below non-dimensional pressure gradient for a gradient wind in cylindrical system of coordinates:

$$\frac{\frac{\partial p}{\partial r}}{\rho\, f\, v_g}$$

$r$ is the distance from the center of the cyclone and $v_g = f(r)$.

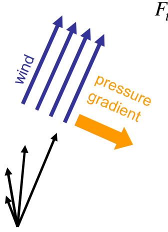
b) What is the magnitude of the non-dimensional pressure gradient for a geostrophic wind?

To the solution

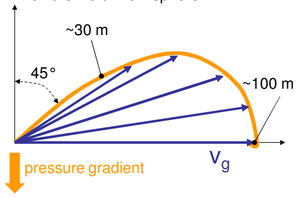
# Ekman spiral

Wind direction changes rapidly at the top of the boundary layer (in the outer layer). When approaching the ground the wind direction turns towards the decreasing pressure. The phenomenon is described by a model similar to the geostrophic wind model, but forces rising from the turbulent stresses must be taken into account:

$$\vec{F}_{turb} = \frac{\partial}{\partial z}\left(v_t \frac{\partial u}{\partial z}\right)\vec{i} + \frac{\partial}{\partial z}\left(v_t \frac{\partial v}{\partial z}\right)\vec{j} \quad \left[\frac{N}{m^3}\right]$$




The solution for a medium latitude case on the North hemisphere:



## Velocity magnitude in the boundary layer

Two important effects must be taken into account:
**surface roughness** and **thermal stratification**.

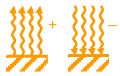
The Monyin-Obuhov profile:

$$\frac{u}{u_*}=\frac{1}{\kappa}\left(ln\frac{z}{z_0}+\beta\frac{z}{L}\right)$$

$z_0$: roughness height;
$\kappa$: Von Kármán const;
L: M-O scale.

$$L=-\frac{u_*^2}{\kappa\frac{g}{T}\frac{H_0}{\rho c_p}}$$

$H_0$: heat flux
($H_0$ ~ -dΘ/dz)




The profile of a constant density flow past a flat plate for comparison:

$$\frac{u}{u_*}=\frac{1}{\kappa}ln\left(\frac{9\,z\,u_*}{\nu}\right)$$

We can note that, the Reynolds number does not count in atmospheric flows.

## Summary

Most important atmospheric flow related physical phenomena beyond the scope of basic level fluid mechanics:

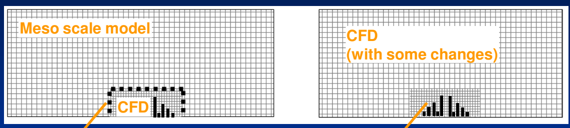
We have touched upon these topics

- Thermal stratification;
- Adiabatic compression and expansion due to vertical flows;
- Variation of density in vertical flows due to the hydrostatic pressure;
- Coriolis force;
- Turbulence in stratified medium;
- Moisture transport and phase changes;
- Surface energy balance involving the radiation heat transport, the heat storage and a number of other complex phenomena.

# CFD based atmospheric simulations

Gergely Kristóf Ph.D., Miklós Balogh, Norbert Rácz
29-th March 2009.

## Advantages of a CFD based model




model conversion interface

- Either the surface geometry is described in high details or (alternatively) meso-scale effects are taken into account.
- Bidirectional interface is a source of numerical errors eg. it can cause partial reflection.

grid refinement

- Higher accuracy
- No limits on geometrical precision
- Flexible meshing
- Advanced turbulent models
- Easy customization
- Advanced pre- and post processing tools

## Methodology

ANSYS FLUENT
+ transformation system
+ customized source terms

## Mathematical description

$$\tilde{\rho} = \rho_0 - \rho_0 \beta(\tilde{T} - T_0)$$

$$\nabla \cdot \tilde{\mathbf{v}} = 0$$

$$\frac{\partial}{\partial t}(\rho_0 \tilde{\mathbf{v}}) + \nabla \cdot (\rho_0 \tilde{\mathbf{v}} \otimes \tilde{\mathbf{v}}) = -\nabla \tilde{p} + \nabla \cdot \boldsymbol{\tau} + (\tilde{\rho} - \rho_0)\mathbf{g} + \mathbf{F}$$

$$\frac{\partial}{\partial t}(\rho_0 c_p \tilde{T}) + \nabla \cdot (\tilde{\mathbf{v}} \rho_0 c_p \tilde{T}) = \nabla \cdot (K_t \nabla \tilde{T}) + S_T$$

$$\frac{\partial}{\partial t}(\rho_0 k) + \nabla \cdot (\rho_0 \tilde{\mathbf{v}} k) = \nabla \cdot \left(\frac{\mu_t}{\sigma_k} \nabla k\right) + G_k + G_b - \rho_0 \varepsilon + S_k$$

$$\frac{\partial}{\partial t}(\rho_0 \varepsilon) + \nabla \cdot (\rho_0 \tilde{\mathbf{v}} \varepsilon) = \nabla \cdot \left(\frac{\mu_t}{\sigma_\varepsilon} \nabla \varepsilon\right) + \rho_0 C_1 S \varepsilon - \rho_0 C_2 \frac{\varepsilon^2}{k + \sqrt{\nu \varepsilon}} + C_{1\varepsilon} \frac{\varepsilon}{k} C_{3\varepsilon} G_b + S_\varepsilon$$

Customized volume sources

Transformed variables $\tilde{\rho}, \tilde{T}, \tilde{p}, \tilde{\mathbf{v}}, \tilde{z}$

## Equilibrium profiles

up to the height of 11 km

$\bar{T} = T_0 - \gamma z$

$\bar{p} = p_0 \left( \frac{T_0 - \gamma z}{T_0} \right)^{\frac{g}{R\gamma}}$

$\bar{\rho} = \rho_0 e^{-\zeta z}$

$T_0 = 288.15\,K$

$\gamma = 0.65°C/100\,m$

$p_0 = 1.01325 \cdot 10^5\,Pa$

$g/(R\gamma) = 5.2553$

$\rho_0 = 1.225\,kg/m^3$

$\zeta = 10^{-4}\,m^{-1}$

Standard ISA profile

Approximate profile

Error bound is within 0.4% below 4000 m.

## Transformation expressions

$T = \tilde{T} - T_0 + \bar{T}$

$p = \frac{\bar{\rho}}{\rho_0} \cdot \tilde{p} + \bar{p} = e^{-\zeta z} \cdot \tilde{p} + \bar{p}$

$\rho = \tilde{\rho} - \rho_0 + \bar{\rho}$

$z = -\frac{1}{\zeta} Ln(1 - \zeta \tilde{z})$

$w = \frac{\rho_0}{\bar{\rho}} \tilde{w} = \tilde{w} e^{\zeta z}$

## Summary of source terms

In momentum equation:

$S_u = \rho_0 f v - \rho_0 \ell \tilde{w} J$

$S_v = -\rho_0 f u$

$S_w = \rho_0 (J^2 - 1)(\ell u J^{-1} + \beta(\tilde{T} - T_0) g) + \rho_0 \ell u J^{-1} + \zeta\, J (\tilde{p} - \rho_0 \tilde{w}^2)$

In the energy equation:

$S_T = J\, S_\Theta - \rho_0 c_p \tilde{w} (\Gamma - \gamma) J$

In the transport equation of turbulent kinetic energy

$S_k = -\beta g \frac{\mu_t}{Pr_t} (\Gamma - \gamma)$

In turbulent dissipation equation:

$S_\varepsilon = -C_{1\varepsilon} C_{3\varepsilon} \frac{\varepsilon}{k} \beta g \frac{\mu_t}{Pr_t} (\Gamma - \gamma)$

$\ell = 2\Omega \cos\phi$

$f = 2\Omega \sin\phi$

$J = (1 - \zeta \tilde{z})^{-1}$

# Related publications


[1] **Kristóf G, Rácz N, Balogh M**: Adaptation of Pressure Based CFD Solvers for Mesoscale Atmospheric Problems, *Boundary-Layer Meteorol, 2008.*

[2] **N.Rácz, G.Kristóf, T.Weidinger, M.Balogh**: Simulation of gravity waves and model validation to laboratory experiments, *CD, Urban Air Quality Conf. Cyprus, 2007.*

[3] **G.Kristóf, N.Rácz, M.Balogh**: Adaptation of pressure based CFD solvers to urban heat island convection problems, *CD, Urban Air Quality Conf. Cyprus, 2007.*

[4] **G.Kristóf, N.Rácz, Tamás Bányai, Norbert Rácz:** Development of computational model for urban heat island convection using general purpose CFD solver, *ICUC6, 6-th Int.Conf.on Urban Climate, Göteborg, pp. 822-825., 2006.*

[5] **G. Kristóf, T. Weidinger, T. Bányai, N. Rácz, T.Gál, J.Unger**: A városi hősziget által generált konvekció modellezése általános célú áramlástani szoftverrel - példaként egy szegedi alkalmazással, *III. Magyar Földrajzi Konferencia, Budapest, 2006.*, Bp, CD

[6] **Kristóf G., Rácz N., Bányai T., Gál T., Unger J., Weidinger T.:** A városi hősziget által generált konvekció modellezése általános célú áramlástani szoftverrel– összehasonlítás kisminta kísérletekkel *A 32. Meteorológiai Tudományos Napok előadásai. Országos Meteorológiai Szolgálat, Bp.*, **2006**

[7] **Dr. Lajos T., Dr. Kristóf G., Dr. Goricsán I., Rácz N.:** Városklíma vizsgálatok a BME Áramlástan Tanszékén, hősziget numerikus szimulációja *VAHAVA projekt (A globális klímaváltozás: hazai hatások és válaszok) zárókonferenciája Bp. CD,* **2006**

[8] **Rácz N. és Kristóf G.**: Hősziget cirkuláció kisminta méréseinek összehasonlítása saját fejlesztésű LES modellel *Egyetemi Meteorológiai Füzetek No. 20 ELTE Meteorológiai Tanszék, Bp. 173-176,* **2006.**

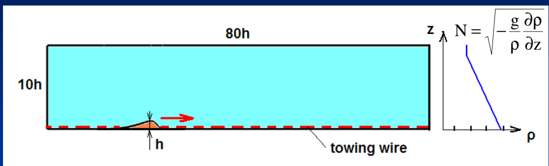
[9] **M. Balogh, G. Kristóf**:Automated Grid Generation for Atmospheric Dispersion Simulations, *pp.1-6., MICROCAD konferencia, Miskolc, 2007.*


# Two validation examples

Comparison with the results of water tank experiments

# Experimetal setup

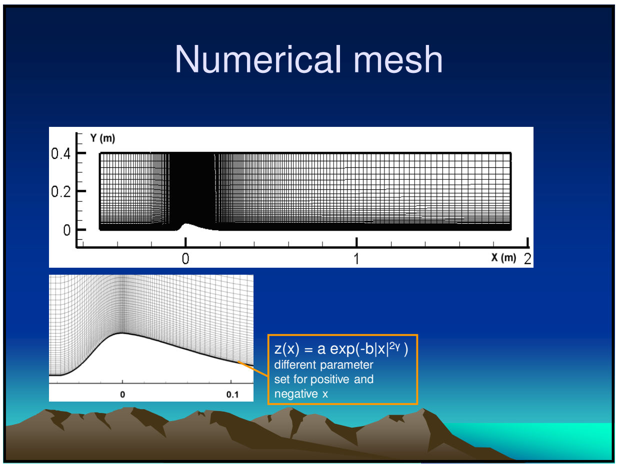
Gyüre, B. and Jánosi, I.M., 2003. Stratified flow over asymmetric and double bell-shaped obstacles. *Dynamics of Atmospheres and Oceans 37*, 155-170.

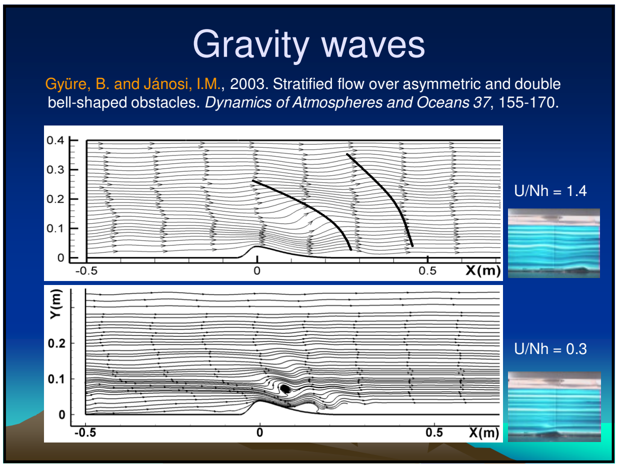


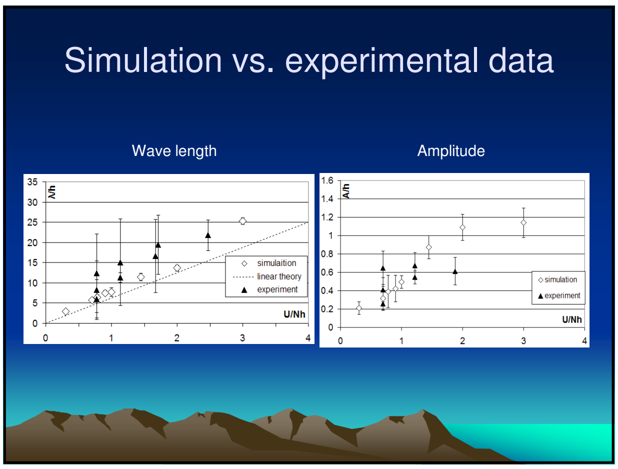

$$N = \sqrt{-\frac{g}{\rho}\frac{\partial \rho}{\partial z}}$$

Uniformly stratified salt water:
ρ= 1.03- 1.00 g/cm$^3$
Typical towing speeds:
U = 1-15 cm/s

Brunt-Vaisala frequency range:
N= 1.09-1.55 1/s
$Re_{exp}$ ≈ $10^2$-$10^3$
Studied obstacle heights:
h= 20mm, 40mm

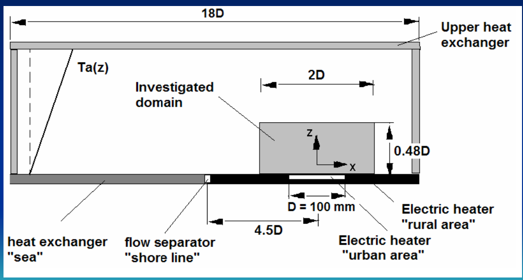

Numerical mesh
Y (m)
0.4
0.2
0
0
1
X (m) 2
0
0.1
z(x) = a exp(-b|x|2γ )
different parameter
set for positive and
negative x


Gravity waves
Gyüre, B. and Jánosi, I.M., 2003. Stratified flow over asymmetric and double bell-shaped obstacles. Dynamics of Atmospheres and Oceans 37, 155-170.
0.4
0.3
0.2
0.1
0
-0.5
0
0.5
X(m)
U/Nh = 1.4
Y(m)
0.2
0.1
0
-0.5
0
0.5
X(m)
U/Nh = 0.3


Simulation vs. experimental data
Wave length
Amplitude
λ/h
simulaition
linear theory
experiment
U/Nh
A/h
simulation
experiment
U/Nh

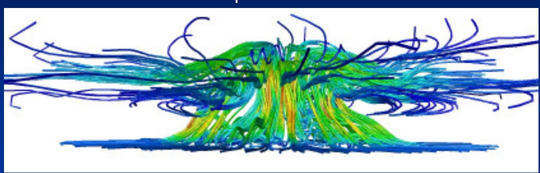
# Experimental setup

A.Cenedese, P.Monti: Interaction between an Inland Urban Heat Island and a Sea-Breeze Flow: A Laboratory Study, 2003.



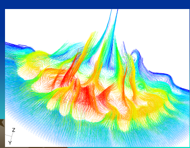

# Large eddy simulation in FLUENT

920 k prismatic cells



Streamlines colored by velocity magnitude

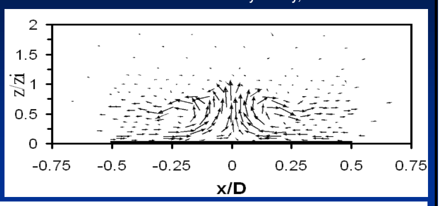
Fine structure of the thermal boundary layer



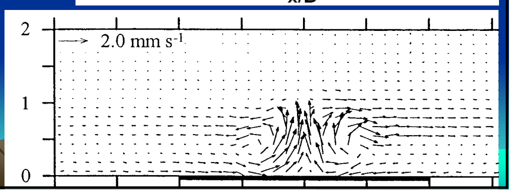
# Thermal convection

A.Cenedese, P.Monti: Interaction between an Inland Urban Heat Island and a Sea-Breeze Flow: A Laboratory Study, 2003.

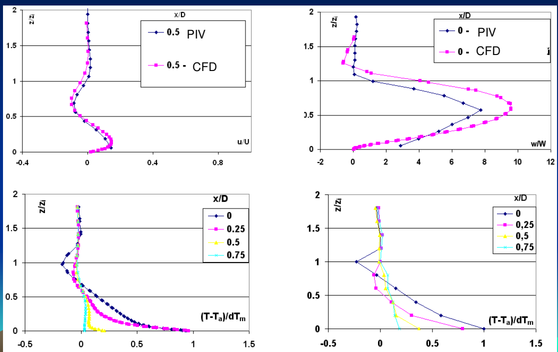
CFD results




PIV results (Cenedese & Monti 2003)



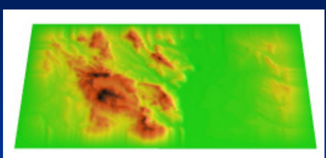
## Comparison of velocity and temperature profiles



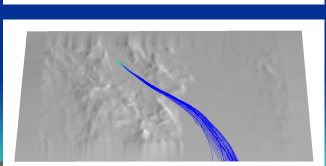

## Some more application examples

Full scale simulations

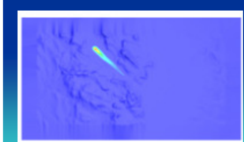
## Meso scale atmospheric dispersion



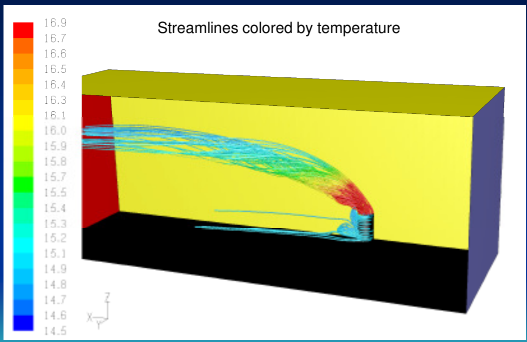
Orography of Pilis mountain



Evolution of surface concentration



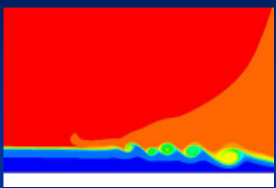
## Micro-scale atmospheric dispersion




Chimney height 180 m
Standard (stable) temperature profile

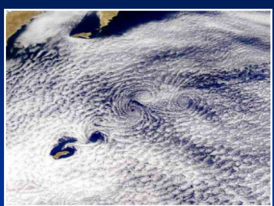
Wind speed: 3m/s
Injection velocity: 5 m/s

## Kelvin-Helmholtz instability

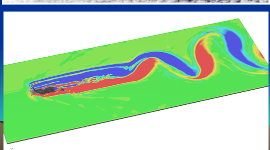


Com domain: 25 km x 5.5 km
Temperature difference 20 °C

## Von Kármán vortices behind a volcanic island



Satellite image about Guadalupe island



First CFD results

# Targeted application areas

- Local circulation modeling:
  - Urban heat island convection, ventilation of cities;
  - See breeze;
  - Valley breeze.
- Power generation and pollution control:
  - Assessment of wind power potential, optimization of wind farms;
  - Plumes emitted by cooling towers and chimneys;
  - Dispersion of pollutant in the urban atmosphere.
- Research of meteorological phenomena:
  - Gravity waves;
  - Cloud formation;
  - Flow around high mountain.
- Simulation of disasters:
  - Large scale fires (e.g. in forest fires or town fires);
  - Volcanic plumes.