Turbulence modelling III.

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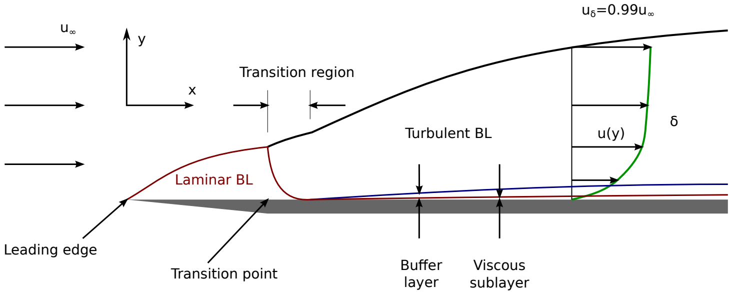
2017.

Wall boundary conditions

- Both k and ε or ω require boundary conditions at the walls
- Before introducing the boundary conditions and the approximate boundary treatment techniques, some theory about wall boundary layers is required.

Boundary Layer



Channel flow

Characteristics

- Flow between two infinite plates $\Rightarrow$ fully developed
- Channel half width: δ
- Bulk velocity: $U_b \stackrel{\text{def}}{=} \frac{1}{\delta}\int_0^\delta \overline{u}\,\mathrm{d}y$
- Bulk Reynolds number: $Re_b \stackrel{\text{def}}{=} \frac{U_b 2\delta}{\nu}$
- $Re_b > 1800$ means turbulence

Channel flow (contd.)

Streamwise averaged momentum equation:

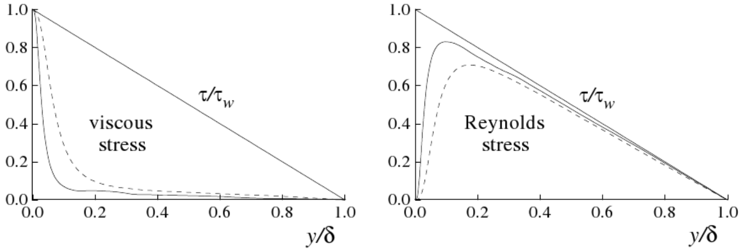
$$0 = \underbrace{\nu \mathrm{d}^2_{y^2} \overline{u}}_{\mathrm{d}_y \tau_l} - \underbrace{\mathrm{d}_y \overline{u'v'}}_{\mathrm{d}_y \tau_t} - \frac{1}{\rho} \partial_x \overline{p} \tag{1}$$

The pressure gradient ($\mathrm{d}_x \overline{p_w}$) is balanced by the two shear stresses: $\tau = \tau_l + \tau_t$
Its distribution is linear:

$$\tau(y) = \tau_w \left(1 - \frac{y}{\delta}\right) \tag{2}$$

Channel flow (contd.)

Two type of shear stresses



The two shear stresses

- The viscous stress is dominant at the wall
- Turbulent stress is dominant far from the wall
- Both stresses are important in an intermediate region

Two scales of the flow at the wall

Definitions

- Friction velocity: $u_\tau \overset{\text{def}}{=} \sqrt{\frac{\tau_w}{\rho}} = \sqrt{-\frac{\delta}{\rho}\mathrm{d}_x\overline{p_w}}$
- Friction Reynolds number: $Re_\tau \overset{\text{def}}{=} \frac{u_\tau \delta}{\nu} = \frac{\delta}{\delta_\nu}$
- Viscous length scale: $\delta_\nu \overset{\text{def}}{=} \frac{\nu}{u_\tau}$

General law of the wall can be characterised:

$$\mathrm{d}_y\overline{u} = \frac{u_\tau}{y}\Phi\left(\frac{y}{\delta_\nu}, \frac{y}{\delta}\right) \tag{3}$$

Φ is a function to be determined!

Law of the wall

In wall proximity

It can be assumed that only the wall scale plays a role in the near wall region:

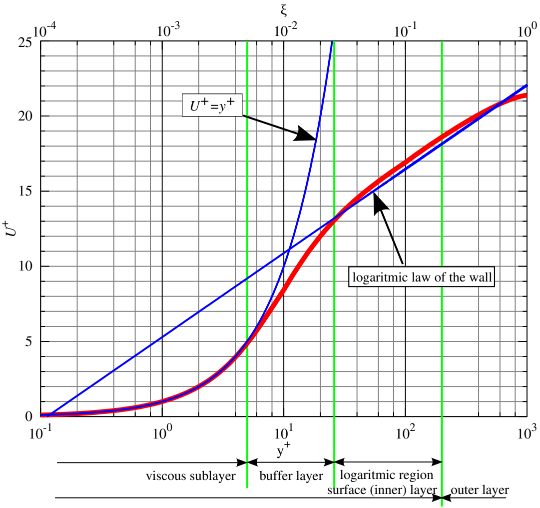
$$\mathrm{d}_y \overline{u} = \frac{u_\tau}{y} \Phi_I\left(\frac{y}{\delta_\nu}\right) \qquad \text{for } y \ll \delta \tag{4}$$

Wall non-dimensionalisation $\square^+$

$$u^+ \overset{\text{def}}{=} \frac{\overline{u}}{u_\tau} \tag{5}$$

$$y^+ \overset{\text{def}}{=} \frac{y}{\delta_\nu} = \frac{y u_\tau}{\nu} \tag{6}$$

Határréteg sebességmegoszlása



Law of the wall

Velocity

Viscous sub-layer

- Only τ_l counts
- $u^+ = y^+$
- for $y^+ < 5$

Logarithmic layer

- Viscosity is not in the scaling
- $\Phi_I = \frac{1}{\kappa}$ for $y \ll \delta$ and $y^+ \gg 1$
- Log-law: $u^+ = \frac{1}{\kappa} \ln(y^+) + B$
  - From measurements: $\kappa \approx 0.41$ and $B \approx 5.2$

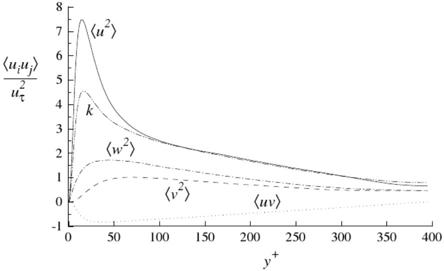
Law of the wall
Velocity

Outer layer

- Φ depends only on y/δ
- In CFD we want to compute it for the specific cases! $\Rightarrow$ We do not deal with it.

Reynolds stress tensor at the wall

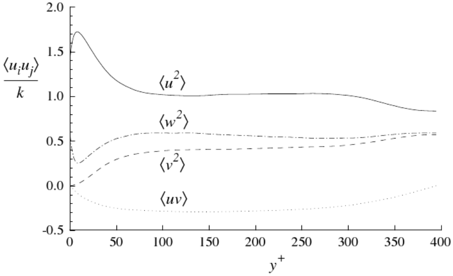
u_τ scaling



Sharp peaks around $y^+ = 20$

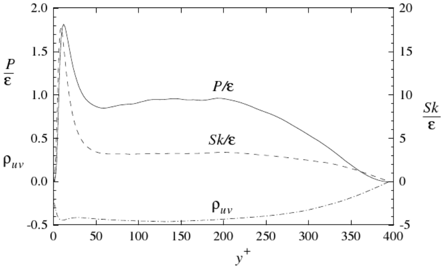
Reynolds stress tensor at the wall

k scaling



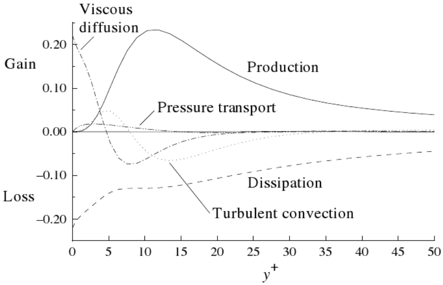
A platau is visible in the log law region.

TKE budget at the wall



- $\mathcal{P}/\varepsilon \approx 1$ in the log-law region
- $\mathcal{P}/\varepsilon \approx 1.8$ close to the wall

TKE budget at the wall



- Turbulence is mainly produced in the buffer region $(5 < y^+ < 30)$
- Turbulence is viscous diffused to the wall
- Turbulence is strongly dissipated at the wall
- Conclusion: $\varepsilon = \nu \mathrm{d}^2_{y^2} k \quad @\ y = 0$

Numerical treatment of the wall layer, actual BC's

Low Re treatment

In this treatment the complete boundary layer is resolved numerically

When to do?

- Low Reynolds number flow, where resolution is feasible
- If boundary layer is not simple, can not be described by law of the wall

How to do?

- Use a turbulence model incorporating near wall viscous effects
- Use appropriate wall resolution ($y^+ < 1$)

Wall functions for RANS in practice (U, ν_t)

- Velocity at the wall:
  - Dirichlet BC, no slip: $U\left(y=0\right)=0$
- Turbulent viscosity (for the wall adjacent cells):
  - Wall shear-stress (friction velocity) by definition:

$$\tau_w = u_\tau^2 = \nu_t \frac{\partial U}{\partial y}$$

  - Laminar case ($y^+ \leq y^+_{lam}$):

$$U^+ = y^+ \rightarrow \nu_t = 0$$

  - Turbulent case ($y^+ > y^+_{lam}$):

$$U^+ = \kappa \ln\left(Ey^+\right) \rightarrow \nu_t = \nu \left(\frac{\kappa y^+}{\ln\left(Ey^+\right)} - 1\right)$$

Wall functions for RANS in practice (k, ϵ, P_k)

- Turbulent kinetic energy (for the wall):
  - Neumann BC: $\frac{\partial k}{\partial y} = 0$
- Turbulent kinetic energy dissipation (for the wall adjacent cells):
  - Equilibrium assumption:

$$P_k = \nu_t \left(\frac{\partial U}{\partial y}\right)^2 = C_\mu \frac{k^2}{\epsilon} \left(\frac{\partial U}{\partial y}\right)^2 = \epsilon$$

  - Implementation:

$$\epsilon = \frac{C_\mu^{0.75} k^{1.5}}{\kappa y} \text{ and } P_k = (\nu + \nu_t) \left|\frac{\partial U}{\partial y}\right| \frac{C_\mu^{0.25} k^{0.5}}{\kappa y}$$

Numerical treatment of the wall layer, actual BC's

High Re treatment

In this treatment the first cell incorporates the law of the wall

When to do?

- High Reynolds number flow, where it is impossible to resolve the near wall region
- If boundary layer is simple, can be well described by law of the wall

How to do?

- Use a turbulence models containing law of the wall BC
- Use appropriate wall resolution $(30 < y^+ < 300)$

The mixture of the two methods is developed:

- to enable the engineer not to deal with the wall resolution
- usually the mixture of the two method is needed, depending on actual position in the domain

Resolution requirements

At any kind of treatment the boundary layer thickness (δ) has to resolved by ≈ 20 cells to ensure accuracy.

Simulation

In the simulation the turbulence phenomena is actually resolved by a numerical technique, by solving the describing equations

Modelling

In the modelling of turbulence the effects of turbulence are modelled relying on theoretical and experimental knowledge. In the computation a reduced description of turbulence is carried out

Direct Numerical Simulation = DNS

The NS equations (describing completely the turbulence phenomena) are solved numerically

Difficulties

- The scales where the dissipation is effective are very small
  - The size of the smallest scales are Reynolds number dependent
- Simulation is only possible for academic situations (e.g.: HIT on 64 $\cdot 10^9$ cells)

Concept of LES

Compromise between RANS and DNS

- RANS: feasible but inaccurate
- DNS: accurate but infeasible

The large scales are import to simulate

- The large scales of the turbulent flow are boundary condition dependent, they needs to be simulated
- The small scales of turbulence are more or less universal and can be modelled 'easily'
- The removal of the small scales form the simulation reduce the computational cost remarkably

Filtering

How to develop the equations?
How to separate between large and small scales?

Spatial filtering, smoothing using a kernel function

$$\langle \varphi \rangle (x_j, t) \stackrel{\text{def}}{=} \int_V G_\Delta(r_i; x_j) \quad \varphi(x_j - r_i, t) \mathrm{d}r_i \qquad (7)$$

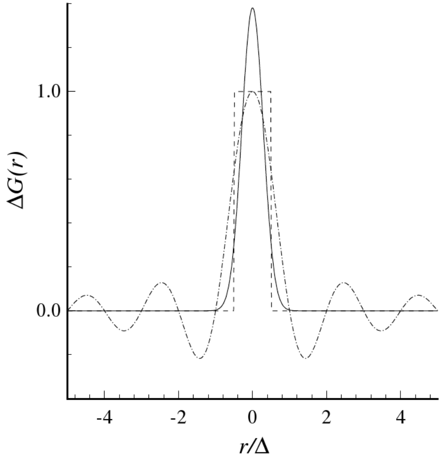
Filtering kernel

- G_Δ is the filtering kernel with a typical size of Δ.
- G_Δ has a compact support (its definition set where the value is non-zero is closed) in its first variable
- To be the filtered value of a constant itself it has to be true:

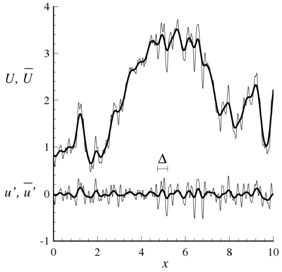
$$\int_V G_\Delta(r_i; x_j)\mathrm{d}r_i = 1 \qquad (8)$$

- If $G_\Delta(r_i; x_j)$ is homogeneous in its second variable and isotropic in its first variable than $G_\Delta(|r_i|)$ is a function of only one variable

Filtering kernel
Examples



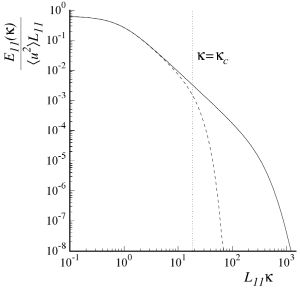
Filtering
Physical space



Fluctuation:

$$\widetilde{\varphi} \stackrel{\text{def}}{=} \varphi - \langle \varphi \rangle \quad (9)$$

$\langle \widetilde{\varphi} \rangle \neq 0$, a difference compared to Reynolds averaging



Recall: the cutting wavenumber (κ_c), below which modelling is needed

Filtered equations

- If using the previously defined (homogeneous, isotropic) filter
- Averaging and the derivatives commute (exchangeable)

$$\partial_i \langle u_i \rangle = 0 \tag{10}$$

$$\partial_t \langle u_i \rangle + \langle u_j \rangle \partial_j \langle u_i \rangle = -\frac{1}{\rho} \langle p \rangle + \nu \partial_j \partial_j \langle u_i \rangle - \partial_j \tau_{ij} \tag{11}$$

- 3D (because turbulence is 3D)
- unsteady (because the large eddies are unsteady)

Sub Grid Scale stress

τ_{ij} is called Sub-Grid Scale stress SGS from the times when filtering was directly associated to the grid

$$\tau_{ij} \stackrel{\text{def}}{=} \langle u_i u_j \rangle - \langle u_i \rangle \langle u_j \rangle \tag{12}$$

- It represents the effect of the filtered scales
- It is in a form a stress tensor
- Should be dissipative to represent the dissipation on the filtered small scale

Eddy viscosity model

- Same as in RANS
- $$\tau_{ij} - \frac{1}{3}\tau_{kk}\delta_{ij} = -2\nu_t \langle s_{ij} \rangle \tag{13}$$
- Relatively a better approach since the small scales are more universal
- Dissipative if $\nu_t > 0$.

Smagorinsky model

- $$\nu_t = (C_s \Delta)^2 |\langle S \rangle| \tag{14}$$
- $$|\langle S \rangle| \stackrel{\text{def}}{=} \sqrt{2 s_{ij} s_{ij}} \tag{15}$$
- C_s Smagorinsky constant to b determined
  - using spectral theory of turbulence
  - using validations on real flow computations
- Δ to be prescribed
  - Determine the computational cost (if too small)
  - Determine the accuracy (if too big)
  - 80% of the energy is resolved is a compromise

Scale Similarity model

Let us assume that the cuted small scales are similar to the kept large scales!
A logical model:

$$\tau_{ij} \stackrel{\text{def}}{=} \langle\langle u_i\rangle \langle u_j\rangle\rangle - \langle\langle u_i\rangle\rangle \langle\langle u_j\rangle\rangle \tag{16}$$

Properties

- It is not dissipative enough
- It gives feasible shear stresses (from experience)
- Logical to combine with Smag. model!

Dynamic approach

- The idea is the same as in the scale similarity model
- The theory is more complicated
- Any model can be made dynamic
- Dynamic Smagorinsky is widely used (combining the two advantages)

Numerical issues

- The spatial numerical schemes are used on the border of their capabilities (wave length = cell siez), when $Delta = h$ (h = cell size)
- The numerical schemes remarkably influence the result
- Grid in-dependency as a function of h/Δ: practically impossible

Boundary Conditions
Periodicity

- Periodicity is used to model infinite long domain
- The length of periodicity is given by the length scales of turbulence

Boundary Conditions
Inlet

- Much more difficult than in RANS
- Turbulent structures should be represented
  - Vortices should be synthesized
  - Separate precursor simulation to provide „real'' turbulence

Boundary Conditions
Wall

$$y^+ \approx 1 \tag{17}$$

$$x^+ \approx 50 \tag{18}$$

$$z^+ \approx 10-20 \tag{19}$$

Results

Time averaged quantities

- Can be used similarly as results of RANS
- In a lucky situation it is more accurate than the RANS result, in case of bad use can be much more inaccurate

Results

Instantaneous structures

- The movement of the vortices can be tracked
- Enables the control of turbulence

Kérdések

Thanks for your attention!