Extended formulation of an enhanced k-ε model for realizable boundary conditions

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ABSTRACT

Atmospheric flow simulations in engineering practice – such as simulations of pollutant dispersion, wind load on buildings, wind climate of urban areas – are fairly sensitive to the inlet boundary conditions. The objective of the present paper is to introduce suitable profiles for velocity and turbulent quantities, which do not only fit the measured vertical distributions, but also guarantee the overall consistency of the turbulence model. If the latter aspect is not verified, the inlet profiles deteriorate rapidly the area of investigation exposed to unrealistic wind profiles. Considering that the atmospheric boundary layer profiles are not solutions of the model equations implemented in general purpose CFD solvers, the models should be corrected via additional source terms. In former studies, Parente et al. (2011a, b) proposed a formulation – including source terms in the transport equations of the standard k-ε model – that guaranties consistency between the model and inlet profiles. In the present study, the formulation is further extended to improve the stability of the approach as well as the realistic character of the produced profiles.

1 INTRODUCTION

The application of two-equation RANS turbulence models with standard rough wall functions often results in unsatisfactory predictions, due to an inconsistent formulation of the law of the wall for rough surfaces and the fully developed inlet conditions for ABL simulations, see e.g. Richards and Hoxey (1993), Blocken et al. (2007a), Blocken et al. (2007b), Franke et al. (2007), Gorle et al. (2009) and Parente et al. (2011a). It means that the inlet profiles deteriorate throughout the computational domain, namely significant differences appear between the turbulent profiles at the inlet and outlet sections. Recently, Parente et al. (2011b) proposed a new approach ensuring consistency among turbulence model, inlet and wall boundary conditions for the numerical simulation of neutral ABL flows. This is accomplished through a reformulation of the wall function based on the aerodynamic roughness and on the derivation of the kinetic energy inlet profile from the solution of the turbulent kinetic energy transport equation, with the re-definition of the coefficient C_μ as a wall distance dependent value. Such an approach has been validated for the simulation of a homogeneous neutral ABL and a ground mounted bluff body using the commercial CFD code FLUENT by Ansys Inc. (Parente, 2011b).

2 ENHANCED INLET PROFILES AND SOURCE TERMS

2.1 Turbulent kinetic energy profiles with enhanced agreement with measurements

Although the set of inlet conditions (Eq. 1) proposed by Richards and Hoxey (1993) are widely used for simulating the neutral ABL in engineering applications. These profiles automatically satisfy the 1D k-ε equations with modified turbulent dissipation rate Prandtl number (σ_ε), its constant turbulent kinetic energy profile does not fit well with the measured profiles.

$$U(z) = \frac{u_\tau}{\kappa} \ln\left(\frac{z + z_0}{z_0}\right), k = \frac{u_\tau^2}{\sqrt{C_\mu}}, \varepsilon(z) = \frac{u_\tau^3}{\kappa(z + z_0)} \quad (1)$$

A more realistic boundary condition for *k* was proposed by Yang et al. (2009), who proposed a height dependent profile for turbulent kinetic energy, which takes the form:

$$k(z) = \sqrt{A_{YA} \ln(z + z_0) + B_{YA}} \quad (2)$$

Gorlé et al. (2009) proposed a modification of the constant C_μ and of the turbulent dissipation Prandtl number, σ_ε, which provided an approximate solution to the system of equations, when using the turbulent kinetic energy inlet profile in the form of Eq. 2. In addition to that, Parente et al. (2011a) introduced two source terms in the transport equations for k and ε respectively, to guarantee that the profiles were solutions to the model equations for a neutral 2D ABL.

The comprehensive approach developed by Parente and Benocci (2010), with a novel profile for turbulent kinetic energy (Eq. 3), was derived from the solution of the turbulent kinetic energy transport equation, resulting in a new set of fully-developed inlet conditions for the neutral ABL:

$$k(z) = A_{PB} \ln(z + z_0) + B_{PB} \quad (3)$$

The consistency between the inlet profiles and the standard k-ε model was ensured with the introduction of a universal source term in the transport equation for the turbulent dissipation rate, and of a height dependent C_μ value:

$$S_\varepsilon(z) = \frac{u_\tau^4}{(z + z_0)^2} \left(\frac{(C_{\varepsilon 2} - C_{\varepsilon 1}) C_\mu^{0.5}(z)}{\kappa^2} - \frac{1}{\sigma_\varepsilon} \right) \quad (4)$$

$$C_\mu(z) = \frac{u_\tau^4}{k^2(z)} \quad (5)$$

In some cases, the turbulent kinetic energy profile (Eq. 3), based on two fitting parameters results in unacceptable deviations from the measured one. Therefore, the formulation is extended in the form of a four parameter profile:

$$k(z) = A \ln\left(\frac{z + z_0}{z_0}\right) + B \left(\frac{z + z_0}{z_0}\right)^2 + C \frac{z + z_0}{z_0} + D \quad (6)$$

It should be observed that this formulation reduces to the profiles proposed by Parente and Benocci (2011b) or Richards and Hoxey (1993) with an appropriate choice of the parameters:

$$A = A_{PB}, B = C = 0, D = B_{PB} + A_{PB} \ln(z_0) \quad (4)$$

$$A = B = C = 0, D = \frac{u_\tau^2}{\sqrt{C_\mu}} \quad (5)$$

Although the four parameter profile implies an additional source term in the turbulent kinetic energy equation to ensure the overall consistency of the model, better agreement can be achieved, with respect to the measured values. The S_k and S_ε source terms required for the consistency of the approach are derived from the 1D k-ε equations. The source term S_ε is not modified with respect to the one in Eq 4, while for the turbulent kinetic energy the following source term should be introduced:

$$S_k(z) = -\frac{\kappa u_\tau}{z_0} \left(4B \frac{z + z_0}{z_0} + C \right) \quad (9)$$

2.2 Profiles extended above the boundary layer height and enhanced stability of the model

When the domain is higher than the boundary layer, the profiles proposed by Parente et al. (2011b) are not appropriate above the boundary layer height (δ), thus the vertical coordinate should be limited to δ in their formulation. Being the derivatives of these profiles zero above δ, the equilibrium is not valid anymore, therefore modified source terms should be applied above the boundary layer in order to keep the inlet quantities constant with the height. The source terms over δ turn into Eq. 10 and Eq. 11.

$$S_k(z > \delta) = \frac{u_\tau^3}{\kappa(\delta + z_0)} \tag{10}$$

$$S_\varepsilon(z > \delta) = C_{2\varepsilon}\sqrt{C_\mu}\frac{u_\tau^4}{\kappa^2(\delta + z_0)^2} \tag{11}$$

Moreover, although the definition of C_μ given by Parente et al. (2011a, b) is mathematically consistent with the model, extremely high C_μ values cause convergence problems, when the measured turbulent kinetic energy is too low. To avoid these problems, C_μ is limited with a smooth transition to a maximum value. This smooth limiter implies further modifications on the source terms, but results more stable model behaviour.

3 VALIDATION OF THE EXTENDED FORMULATION

The extended formulation is implemented in OpenFOAM, following the implementation of the comprehensive approach proposed by Parente et al. (2011a, b), and previously validated for the flow over complex terrain (Balogh et al., 2012).

The new extended ABL formulation was validated on 2D laboratory scale cases against experiments. Three different data sets were used for the validation in order to prove the sufficient fit of the extended formulation, furthermore the validity of the derived source terms. The measured data sets were obtained from wind tunnel experiments applied over flat rough surfaces. One of them is the CEDVAL A1-1 case (Leitl ,1998), the latters are flat reference cases of wind tunnel simulations over 2D and 3D hills. The 2D hill flat reference data is obtained by Khurshudyan et al. (1981) and it is accessible via the ERCOFTAC database (case 69). The 3D hill flat reference measurements are obtained in the thermally stratified wind tunnel of The University of Tokyo under neutral conditions, using three-dimensional laser doppler anemometry (Takahashi et al., 2005).

3.1 Geometry and mesh

The simple 2D domains are meshed with uniform longitudinal resolution and increasing cell size towards the wall, where the size of the wall adjacent cell determined from $y^+ \sim 20$. The properties of the domains and meshes are summarized in Table 1, where $\Delta z_{last}/\Delta z_{first}$ denotes the expansion ratio between the first and last cell height.

Table 1: Domain properties for 2D domains of validation cases.

Case	Length [m]	Height [m]	N_x	N_z	$\Delta z_{last}/\Delta z_{first}$
CEDVAL A1-1	5	1	400	74	33.95
ERCOFTAC 69	5	1.6029	400	81	46.67
TOKYO UNI WT	5	0.5	400	32	4.2

3.2 Boundary conditions

The interpolated experimental data of the velocity and turbulent quantities were introduced for boundary condition at inlet and the upper boundary of the domain, while zero longitudinal gradient is imposed at the outlet. The fitting parameters for the extended profiles are summarized in Table 2 and 3. These profile parameters are calculated by an algorithm applies non-linear regression for the velocity, and a non-linear least squares fit using Newton-Raphson method for the turbulent kinetic energy. Both the velocity and the kinetic energy profile fits are built in one automatic framework implemented in FORTRAN 95. Using this framework, the four parameter profile fits provide a significant improvement on the correlation between the measured values and the fitted ones, summarized in Table 4.

Table 2: Velocity profile properties for the validation cases.

Case	u_τ [m/s]	z_0 [m]	δ [m]
CEDVAL A1-1	3.58e-1	7.11e-4	1.00
ERCOFTAC 69	1.82e-1	1.57e-4	1.25
TOKYO UNI WT	8.61e-2	5.94e-4	0.51

Table 3: Turbulent kinetic energy profile parameters for the validation cases.

Case	A	B	C	D	A_{PB}	B_{PB}
CEDVAL A1-1	9.69e-2	1.99e-7	-6.30e-4	2.03e-1	-3.82e-2	5.15e-1
ERCOFTAC 69	1.20e-2	0.33e-8	-0.54e-4	1.14e-1	-5.90e-3	1.15e-1
TOKYO UNI WT	1.37e-2	2.23e-8	-5.69e-5	-7.09e-3	-5.25e-3	4.95e-2

Table 4: Correlation coefficients for the velocity and turbulent kinetic energy profiles.

Case	R_U	R_k (new)	R_k (PB)
CEDVAL A1-1	0.997	0.822	0.119
ERCOFTAC 69	0.999	0.966	0.416
TOKYO UNI WT	0.977	0.941	-0.785

3.3 Simulation results

Figures 1-3 shows inlet and outlet profiles of velocity, turbulent kinetic energy and dissipation rate for the three different cases.

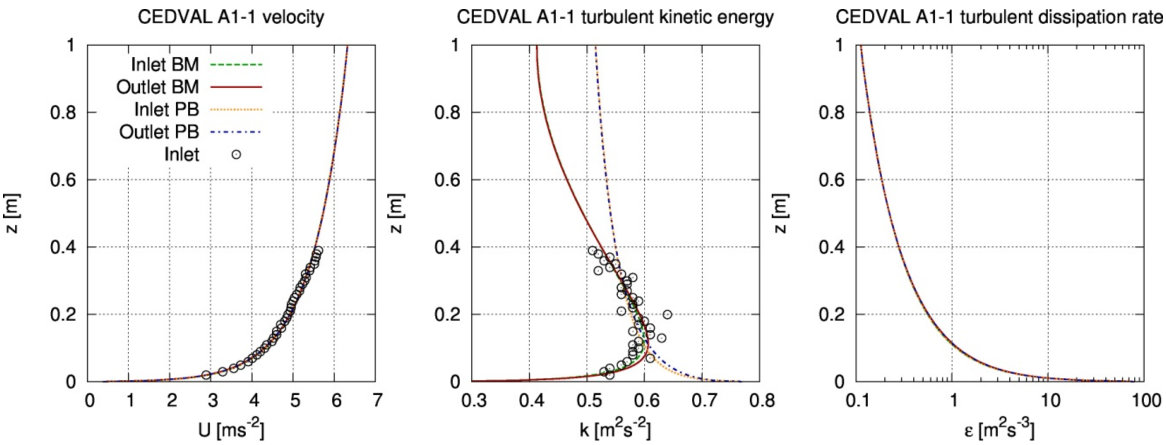

Figure 2: Inlet and outlet profiles against measurements for CEDVAL A1-1 case (PB. – Parente et al. 2011a, BM. – extended formulation).

It can be observed that the extended approach as well as the one proposed by Parente et al. (2011a, b) ensured homogeneity of velocity and turbulence between inlet and outlet sections of the domain. The new turbulent kinetic energy profile formulation with four parameters suitably fit for all measured profiles. For the ERCOFTAC 69 flat reference case (Figure 2), the homogeneity is guaranteed even with limited general C_μ value, as well above the boundary layer height, thanks to the source term transition.

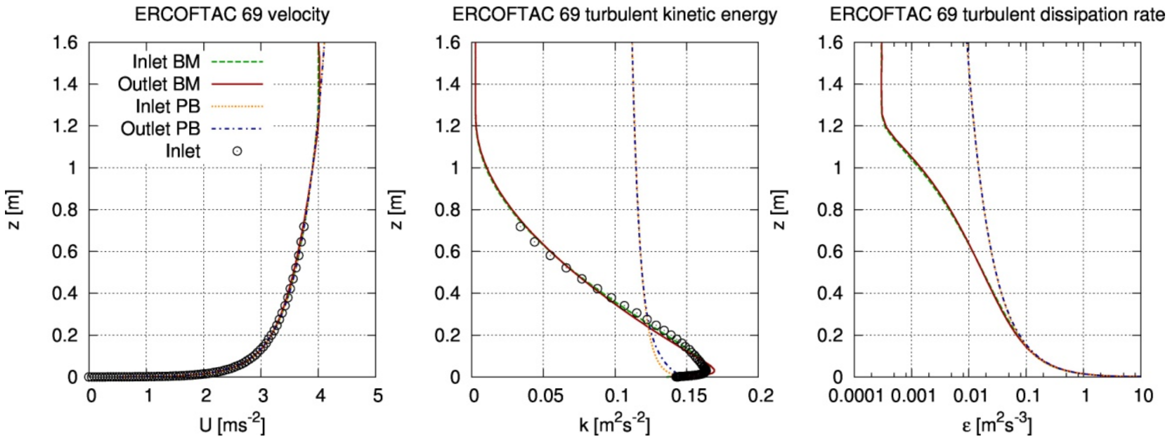

Figure 2: Inlet and outlet profiles against measurements for ERCOFTAC 69 case (PB. – Parente et al. 2011a, BM. – extended formulation).

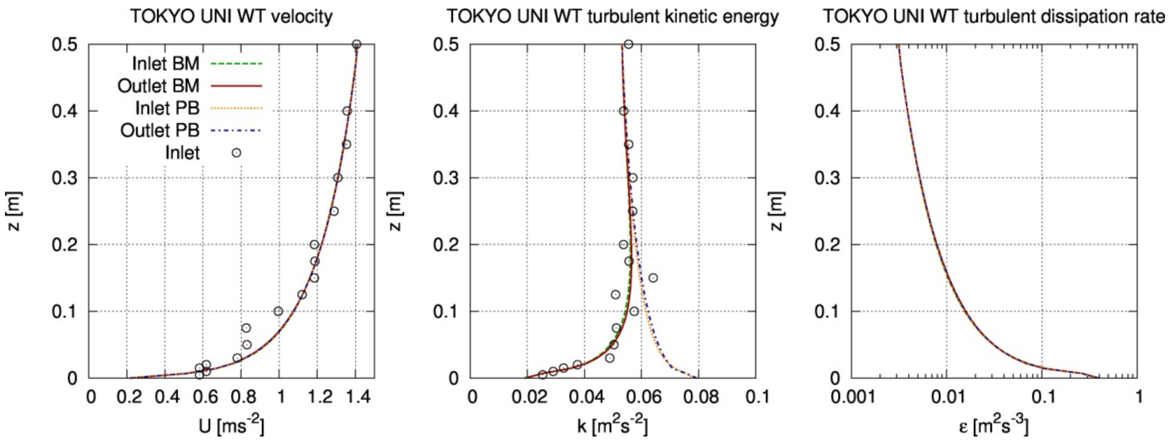

Figure 3: Inlet and outlet profiles against measurements for TOKYO UNI WT case (PB. – Parente et al. 2011a, BM. – extended formulation).

4 CONCLUSIONS

The novel approach proposed by Parente et al. (2011a, b) is improved with a four parameters turbulent kinetic energy profile, to achieve better agreement between the boundary conditions and the experimental profiles, within the boundary layer and above. This implies modifications on the approach, in order to guarantee the stream-wise homogeneity of the fully developed boundary layer. Accordingly, modified source terms are derived for the transport equations of turbulent kinetic energy and dissipation rate. The extended model is validated against experimental data obtained from wind tunnel measurements. Results indicate that the four parameter profile suitably reproduces all measured profiles, as indicated by correlation coefficients above 0.8 for all cases. Furthermore, the extended model ensures the stream-wise homogeneity of the velocity and turbulence profiles at the inlet and outlet sections of the domain, even above the boundary layer thickness.

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